

# Discounting climate change damages: Working note for the Stern review

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Final Draft

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## Overview

This working note provides a basic overview of discounting in the context of climate change policy. After defining the social discount factor and social discount rate in terms of shadow prices (section 2), and noting the limitations of cost-benefit analysis for climate change (section 3), the determination of efficient social discount rates is discussed given: the impact of uncertainty about future economic conditions (section 4.2), the effect of heterogeneous time preferences (4.3) and time inconsistency issues (4.4). Fairness between generations is then discussed (5.1) and some alternatives to using discount factors are considered (5.2). It is concluded that the shadow discount rate should be declining over time to reflect the certainty-equivalent path. It is also argued that the underlying utility discount rate is very small, possibly zero to a first approximation.

**Keywords:** discounting, efficiency, uncertainty, heterogeneous preferences, declining discount rates, time inconsistency, intergenerational equity.

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\*The opinions expressed are Cameron's, and are not all shared by Paul. Paul had no input into later revisions. Cameron takes all responsibility for errors or omissions.

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## 1. Introduction

How much is it worth investing today to prevent damages from climate change in the distant future? The theoretical machinery needed to answer this question has, largely, already been developed because the appraisal of government policies, projects and programmes almost always involves trading off costs now in return for benefits in the future.<sup>1</sup> Economic appraisal requires a *price* to convert costs and benefits at different points in time into common units. In social cost-benefit analysis, this price is the social *discount factor*,  $D(t)$ .

## 2. Shadow discount factors and discount rates

Social discount factors are prices of future consumption relative to consumption today. They are used to convert flows of future cost and benefits into present equivalents. Many prices are revealed by an appropriate market, and a relative price of future consumption could be calculated from risk-free long-term interest rates. There are at least four arguments, however, for the inappropriateness of simply using market prices.<sup>2</sup>

1. *Market imperfections*. Market prices often give a misleading signal of value as a result of a sub-optimum distribution of income or because of other distortions in the economy, such as externalities, government taxation, imperfect information and the exercise of market power (Drèze and Stern, 1990). Under such

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<sup>1</sup> Although costs and benefits tend to be assessed from an anthropocentric viewpoint, this is not the only relevant perspective. Many argue that stewardship and the rights of other species are also relevant, independently of whether they are reflected in human preferences. As many economists (e.g. Lind, 1982) have pointed out, cost-benefit analysis — in which discounting features so importantly — is merely a guide for decision making, not a substitute for judgment.

<sup>2</sup> See Sen (1982), who notes that this issue was debated vigorously in the late 1950s and 1960s, and cites 19 papers on the topic.

conditions, market prices do not reflect the “shadow price”, or the true social opportunity cost of the resource.

2. *Super-responsibility.* It is sometimes argued that the government has a responsibility to both current and future generations. As markets only reveal the preferences of the current generation, the government should not rely solely on market information.
3. *Dual-role.* The members of the present generation in their political role may be more concerned about future generations than their day-to-day activities on current markets would reveal.
4. *Isolation argument.* Finally, Sen (1982) has argued, somewhat controversially, that individuals may be willing to join in a collective savings contract, even though they are unwilling to save as much in isolation.

Although some of these positions generated heated argument, the overall view clearly emerged that real risk-free *market* interest rates provide an inappropriate conceptual basis for social discounting.<sup>3</sup> This, of course, does not mean that market interest rates are entirely irrelevant. When public investment crowds out private investment, the opportunity cost of that investment is the market interest rate. However, public expenditure displaces private expenditure to a different extent depending upon the particular investment, and Lind (1982) recommends accounting for crowding-out effects ‘by directly analyzing the magnitude of these effects and the converting them to their consumption equivalents through the use of a shadow price on capital.’<sup>4</sup>

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<sup>3</sup> See Lind (1982) for a clear statement of the consensus view emerging from the influential 1977 conference on the topic. See also Arrow (1995) expressing the same view.

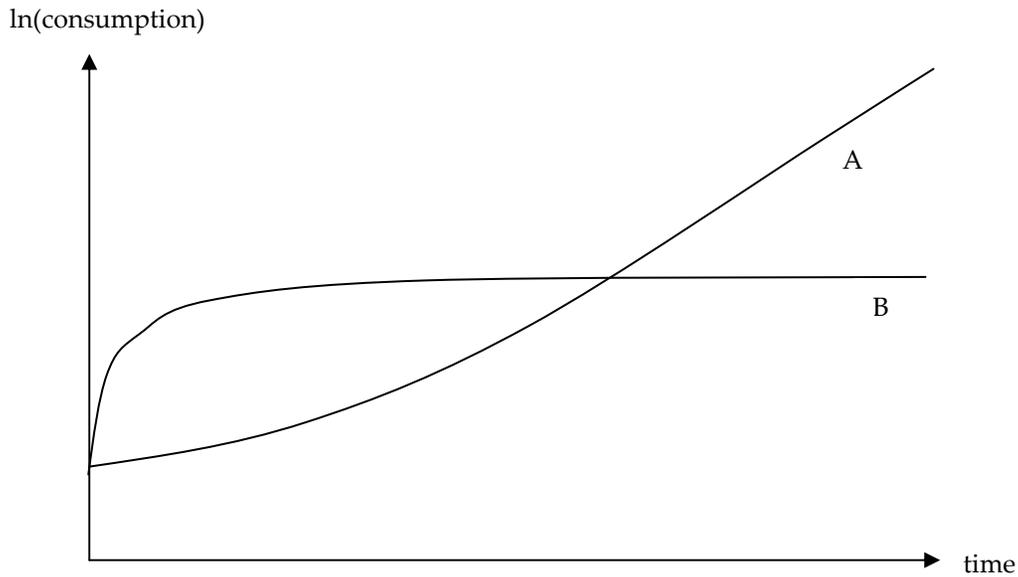
<sup>4</sup> This shadow pricing approach is not currently used in the UK (HM Treasury, 2003), reflecting a mix of practicability and the view that the real risk-free interest rate and the shadow discount rate are quite close in magnitude (Spackman, 1991; Pearce and Ulph 1999). See also Arrow (1995).

This paper focuses on shadow discount factors, which describe the true social opportunity cost of future consumption relative to consumption today. The relative price of future consumption (to current consumption) depends upon the level of future consumption, which in turn depends upon productivity and economic growth rates. For instance, Figure 1 shows two different future consumption paths (expressed as logarithms). The discount factors appropriate for these two paths are very different, because the consumption growth rates along the two paths are very different. The relative price of future consumption (the discount factor) falls as future generations become more wealthy.

More specifically, Path A initially has a slow growth rate, before reaching a high (and constant) consumption growth rate. Because the shadow discount factor is inversely related to future consumption, it initially falls slowly, but then declines more rapidly as consumption growth increases. In contrast, Path B has a very high initial consumption growth rate, but then stagnates at a growth rate not much above zero. Along Path B, the shadow discount factor would initially fall very quickly, as consumption increases, but would then remain relatively constant (because consumption levels are relatively constant).

In other words, shadow discount factors are only applicable *along a particular path*. However, our response to climate change is likely to involve large-scale investment with “non-marginal” impacts, which could shift the economy from one path to another. Under these circumstances, the conventional approach to cost-benefit analysis, which employs discounting as a short-cut, may be inapplicable (see Appendix). If this is so, full welfare analysis, without the short-cut of discounting, would be required. These issues are further discussed in section 3.

Figure 1: Different future growth paths



For marginal investments, it is often convenient to think about the trade off between present and future consumption in terms of the shadow discount rate,  $s(t)$ , which is the annual rate of decline in the shadow discount factor,  $D(t)$ . There is a direct correspondence between the two concepts, and they are connected by the equation:

$$D(t) = \frac{1}{(1 + s(t))^t} \tag{1}$$

Using a constant and positive discount rate implies that the discount factor declines approximately exponentially,<sup>5</sup> implying that cash flows in the future are worth less than cash flows today.<sup>6</sup> There are two reasons why shadow discount factors normally fall over time (which is equivalent to saying that the shadow discount rate is positive). First, people generally prefer to have good things earlier rather than later. Second, because

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<sup>5</sup> The continuous time analogue of the discount factor is the discount function, given by

$$D(t) = \exp\left[-\int_0^t s(\tau) d\tau\right], \text{ and if } s \text{ is constant this is } D(t) = \exp(-st).$$

<sup>6</sup> As the discount factor is a relative price, for convenience it is often set to unity for costs and benefits today. As such, discount factors for the future are typically less than unity.

capital is productive — funds invested tend to yield positive returns — we expect to be better off in the future than we are today. These two reasons are reflected in the recommended approach to social discounting (Lind, 1982, p 89), which is to employ the social rate of time preference, so that the discount rate,  $s$ , is given by:

$$s(t) = \delta + \mu g(t) \tag{2}$$

where  $\delta$  is the rate of time preference (a ‘utility’ discount rate),  $\mu$  is the elasticity of marginal utility and  $g(t)$  is the rate of growth of consumption per capita at time  $t$ .<sup>7</sup> The discount rate,  $s(t)$ , is the rate at which future *consumption* (or cash flows) is discounted. In contrast,  $\delta$  is the rate at which future *utility* (or wellbeing) is discounted. Notice that even if the utility discount rate  $\delta$  is zero — so utility now and utility in the future is given equal weight — the social discount rate is still positive if  $g > 0$  and  $\mu > 0$ .

As Figure 1 indicates, the appropriate social discount factor is a function of the future economic path. In general, the social discount rate,  $s(t)$ , is *not* constant over time, but is a function of the expected future rate of consumption growth,  $g(t)$ . For instance, if it were known with certainty that future consumption growth will be cyclical, then the appropriate social discount rate should vary to reflect those cycles. Equally, if climate change impacts are expected to slow down future economic growth rates, the social discount rate should decline accordingly.<sup>8</sup> In the extreme case where future growth is

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<sup>7</sup> Equation (2) is just an accounting relation for a utility function expressing a preference ordering satisfying the axioms of Koopmans (1960). Here, the social rate of time preference is given by the consumption discount rate, which is the rate at which marginal utility decreases with time, accounting for the impact of changing consumption. Note that societies composed of real people (with heterogeneous preferences) are discussed further in section 3.3 below.

<sup>8</sup> The concept of economic growth refers to growth in the *value* of goods and services provided by the economy, including in non-market sectors, such as services derived from natural capital. Conventional measurements of GDP growth omit (or inadequately measure) non-market sectors upon which climate change may have significant impacts.

negative (so that long-term recessions are anticipated), then the appropriate discount rate could be negative.<sup>9</sup>

### **Estimating the utility discount rate**

Pearce and Ulph (1999) summarise various estimates of the appropriate utility discount rate, dividing it into two components — impatience and life chances. They conclude that the component for impatience (the ‘rate of pure time preference’) lies between zero and 0.5% (best guess 0.3%), although they note that there is ‘no clear view what the rate of pure time preference should be.’<sup>10</sup> Nevertheless, philosophers and many economists have long argued that for social decisions, anything other than a zero rate of pure time preference is unethical.<sup>11</sup> This clearly contradicts human behaviour, which may be better described by the less demanding standards of ‘agent-relative ethics’ (Arrow, 1999). Nevertheless, this ethical perspective for neutrality between generations has a long and fine intellectual pedigree,<sup>12</sup> and seems particularly compelling for long-term challenges such as climate change.

The second component of the rate of time preference — ‘life chances’ — has been defined in different ways. For an individual, the definition is clear: ‘life chances’ reflect the background risk of death which justifies discounting future streams of consumption.

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<sup>9</sup> See Dasgupta et al (1999).

<sup>10</sup> Their estimate is based on Scott (1989), which has been derived from UK savings data. They also argue independently that equity reasons argue against a zero rate of pure time preference, but this does not appear to play a role in the estimation of  $\delta$ . We discuss equity considerations in section 4 below.

<sup>11</sup> See Hepburn (2006) for a review of the arguments.

<sup>12</sup> See Ramsey (1928), Pigou (1932), Harrod (1948), Koopmans (1965), Rawls (1972), Solow (1974) and Broome (1992) among others. In contrast, see Arrow (1995) citing Koopmans (1960, 1964). But whether or not Koopman’s analysis requires a positive utility discount rate, they may need revision after Asheim, Buchholz and Tungodden (2001) and Asheim and Buchholz (2003).

For society, however, definitions vary. Pearce and Ulph (1999) correctly reject the view that it is simply an aggregation of the risk of death for individuals. Instead, they focus upon the 'life chances of whole generations' and calculate the proportion of a generation which will die each year, which they determine to be 1.1%. For social decision-making spanning several generations, however, this may be misguided. The relevant risk, for social decision-making, is the risk of catastrophe eliminating society. As Dasgupta and Heal (1979) argue, 'one might find it ethically feasible to discount future utilities at positive rates, not because one is myopic, but because there is a positive chance that future generations will not exist'.<sup>13</sup> One might speculate that the calculation of the risk of exogenous social collapse would be rather small, probably under 0.5% per annum and possibly 0% to a first approximation (but cf Rees, 2003).

### **3. Problems with cost-benefit analysis and climate policy**

Some commentators argue that social cost-benefit analysis is an inappropriate tool for climate change policy. Criticisms are often based on one of the following reasons: (1) ignoring equity considerations is utterly inappropriate; (2) the welfare economic framework which underpins cost-benefit analysis is inadequate; (3) cost-benefit analysis is appropriate at the margins, but climate change is arguably a non-marginal policy problem.

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<sup>13</sup> Conceptually, there is merit in the view expressed by Broome (1992) that such risks, while 'in a sense...undoubtedly a reason for discounting the wellbeing of future generations', should be accounted for separately, presumably on the ledger. There are good practical reasons, however, for including exogenous risks of calamity that do not vary from project to project in the discount rate, just as an individual would include a component for their personal 'life chances'. Such risks could theoretically be factored into estimates of future growth rates, but it is, in my view, neater to include them as a component in  $\delta$ . Ultimately, wherever such risks are accounted for, the mathematical effect is identical to discounting the future.

First, economics has a tradition of separating efficiency from equity, and social cost-benefit analysis is no exception, where the Kaldor-Hicks criterion is relied upon to justify projects that are efficient.<sup>14</sup> In theory, the distributional effects of a particular policy can be ignored when government can use the tax system to redistribute income to achieve equity. In practice, however, the distributional effects of some projects are important, which is why cost-benefit analysis should be employed as a *guide* for decision making rather than a substitute for judgment (Lind, 1982). For climate policy, distributional effects are arguably paramount, because there is no intergenerational tax system for wealth redistribution (Lind, 1995; 1999). Although economic instruments can create wealth transfers between generations (such as certain changes to tax law and fiscal policy), there is no guarantee that the transfer will reach the intended recipient when there are many intervening generations. In such circumstances, the Kaldor-Hicks criterion appears dubious, and explicit consideration of intergenerational equity is necessary. This does not imply that cost-benefit analysis is pointless, but that results are dangerous if derived without explicit analysis of equity implications.

Second, Sen (1982) has argued that the welfare economic framework is insufficiently robust to deal with questions of intergenerational equity because it fails to incorporate concepts of liberty, rights and entitlements as ends in themselves.<sup>15</sup> In particular, cost-

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<sup>14</sup> A change passes the Kaldor (1939) criterion if the gainers could compensate the losers, and the Hicks (1940) criterion if the losers could not pay the gainers to prevent the change. Compensation is not actually required.

<sup>15</sup> For instance, suppose that torture leaves the person tortured (the 'heretic') worse off and the torturer (the 'inquisitor') better off. If the inquisitor is still worse off than the heretic afterwards, the torture would be justified by cost-benefit analysis, even with equity weights. Sen (1982) argues that society may want to grant the heretic a right to personal liberty that cannot be violated merely to achieve a net gain in utility or an improvement for the worst-off individual. Furthermore, he argues that an analogy between pollution and torture is 'not absurd', and that

benefit analysis does not tend to address questions of rights and responsibilities owed to future generations. These criticisms are important. They further emphasise the fact that within climate policy, cost benefit analysis should be employed as a guide for decision making rather than a substitute for judgment.

Finally, conventional cost-benefit analysis is only applicable for marginal perturbations to the economic path, as proved in the Appendix. For instance, cost-benefit analysis might typically be employed to evaluate a small perturbation around path A in Figure 1, in which case the shadow discount factors for path A would be employed. Cost-benefit analysis is also applicable if we are uncertain whether path A or path B will apply, provided the project concerned is still a marginal perturbation around the future path (whichever is appropriate). In this case, by applying probabilities to paths A and B, certainty-equivalent discount factors could be determined and employed.

However, large scale changes in climate policy could arguably shift the economic path from B to A. This is clearly non-marginal. Under such circumstances, conventional cost-benefit analysis is inapplicable, in so far as a positive NPV would not guarantee an increase in social welfare (see Appendix). Nevertheless, provided a utility discount rate can be specified, the merit of a non-marginal climate policy intervention can be assessed by comparing the stream of social welfare with and without the intervention, as in equation (3) of the Appendix. Although the climate policies implemented by governments so far would probably satisfy the marginality condition, given the scale of challenge, future interventions may need to be assessed by comparing utility streams.

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perhaps the liberty of future generations is unacceptably compromised by the present generation's insouciance about pollution.

## 4. Efficient declining social discount rates

### 4.1 Overview

This section considers three issues: the impact of uncertainty about future economic conditions (section 3.2), the effect of heterogeneous time preferences (3.3) and problems of time inconsistency (3.4).

### 4.2 Uncertainty about future economic conditions

There is substantial scientific and economic uncertainty about climate change impacts, which is compounded by long time horizons. As such, for cost-benefit analysis of climate mitigation policies, certainty-equivalent impacts must be calculated.<sup>16</sup> Similarly (and partly as a result of climate uncertainty) there is also significant uncertainty about future technological progress, economic growth rates, and therefore the appropriate social rate of time preference in the distant future. Suppose that the future comprises two equally likely states with a constant social discount of either 2% or 6%. Discount *factors* corresponding to these two discount *rates* are shown in Table 1. The average of those discount factors is called the ‘certainty-equivalent discount factor’, and working backwards we can determine the ‘certainty-equivalent discount rate’, which starts at 4% and declines asymptotically to 2% as time passes.<sup>17</sup>

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<sup>16</sup> Note that the assumptions underlying cost-benefit analysis may be violated if the investment under consideration will itself produce changes in the future distribution of economic growth rates. In the aggregate, investments in climate change mitigation may do just that, but it seems unlikely that any one investment would significantly alter future economic growth rates. On a separate but related point, note that even if the project is marginal, if it is correlated with economic growth rates, a risk adjustment is required as discussed by Lind (1982). See also the guidance given in Annex 4 of the HM Treasury (2003) Green Book.

<sup>17</sup> The certainty-equivalent average discount rate is given by  $s_c(t) = (1/D_c(t))^{1/t} - 1$ , where  $D_c(t)$  is the certainty-equivalent discount factor.

*Table 1: Numerical example of a declining certainty-equivalent discount rate*

<b>Time (years from present)</b>	<b>1</b>	<b>10</b>	<b>50</b>	<b>100</b>	<b>200</b>	<b>400</b>
Discount factor for 2% rate	0.98	0.82	0.37	0.14	0.02	0.00
Discount factor for 6% rate	0.94	0.56	0.05	0.00	0.00	0.00
Certainty-equivalent discount factor	0.96	0.69	0.21	0.07	0.01	0.00
Certainty-equivalent (average) discount rate	4.0%	3.8%	3.1%	2.7%	2.4%	2.2%

The two key assumptions in this example are that the discount rate is uncertain and *persistent*, so that the expected discount rate in one period is correlated with the discount rate the period before. If these two assumptions hold, efficiency considerations require a declining social discount rate (Weitzman 1998, 2001).

The particular shape of the decline is determined by the specification of uncertainty in economic growth. One approach to determining future uncertainty would be to develop a series of scenarios with different forecast pathways for  $\delta$ ,  $\mu$  and  $g$ , estimate probabilities for each scenario, and then determine the corresponding certainty-equivalent discount rate. An alternative approach is to assume that that future uncertainty in discount rates is reflected by the uncertainty in past discount rates. Newell and Pizer (2003) use data on past US interest rates to estimate a reduced-form time series process which is then employed to forecast future rates.<sup>18</sup> The level of persistence in their discount rate forecasts is high enough to generate a relatively rapid decline in the certainty-equivalent discount rate with significant policy implications.<sup>19</sup>

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<sup>18</sup> As discussed in section 2, the real risk-free interest rate is a different concept from the social rate of time preference, upon which the shadow discount rate is based. Nevertheless, Newell and Pizer (2003) argue that interest rates are a reasonable proxy for which long-term data is available.

<sup>19</sup> Econometric tests reported in Groom et al (2004) and applied by Hepburn et al. (2006) suggest that Newell and Pizer (2003) should have employed a state-space or regime-shifting model instead. Nevertheless, their key conclusion remains intact — the certainty-equivalent discount rate declines at a rate that is significant for the appraisal of long-term projects.

This story, while instructive, is somewhat ad hoc because it begins with the assumption of uncertainty in the discount rate — which is a derived rate of change of a price — rather than examining the underlying uncertainty in economic growth rates and the technological progress. In contrast, Gollier (2001, 2002a, b) analyses a richer optimal growth model, where a utility function is specified,<sup>20</sup> and demonstrates that the results described above can still obtain. Under uncertainty, the social discount rate in equation (2) needs to be modified to account for an additional prudence effect:

$$s = \delta + \mu g - \frac{1}{2} \mu P \text{var}(g) \quad (3)$$

where  $P$  is the measure of relative prudence introduced by Kimball (1990). This prudence effect leads to ‘precautionary saving’, reducing the discount rate. Furthermore, if there is no risk of recession and people have decreasing relative risk aversion, the result of this effect is that the optimal social discount rate is declining.

Weitzman (2004) goes one step further and investigates the uncertainty in technological progress. He derives an effect analogous to Gollier’s prudence effects (which he terms the “stochastic smoothing effect”) and finds a further effect when the underlying growth process is itself unknown (the “statistical forecasting effect”). This second effect also lowers the efficient discount rate for events in the far-distant future.

In sum, results from research on uncertainty in future discount factors suggests rather strongly that employing a declining discount rate is necessary for dynamic efficiency. The consequences on the social cost of carbon and climate policy are important and discussed elsewhere (Pearce et al, 2003; Newel and Pizer, 2003; Guo et al., 2006).

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<sup>20</sup> Specifying a utility function implicitly defines a conception of intergenerational equity, and the optimal solution maximises utility, not the net present value of the cash flows as in Weitzman (1998).

### 4.3 The relevance of heterogeneity

The social rate of time preference discussed in equation (2) reflects the assumption that individual preferences (captured by their utility discount rate  $\delta$  and elasticity of marginal utility  $\mu$ ) have already been aggregated to produce a social preference. Of course, this is easier said than done,<sup>21</sup> given that different people have different rates of time preference. Gollier and Zeckhauser (2005) investigate the optimal collective decision policy when individuals have heterogeneous (and constant) utility discount rates.<sup>22</sup> They find that when individuals have decreasing absolute risk aversion preferences,<sup>23</sup> the optimal collective policy is to employ a declining utility discount rate.

This result is derived from two key insights. First, the efficient collective utility discount rate is a weighted average of the individual utility discount rates. The weights are proportional to each individual's tolerance to consumption fluctuations. If an individual cannot tolerate consumption fluctuations, efficiency demands that she have a constant consumption profile — her time preference does not matter. In contrast, when an individual can tolerate consumption fluctuations, it is efficient to take account of his particular time preference. Hence the weight on each individual's discount rate is proportional to their tolerance to consumption fluctuations. Second, it is efficient for the most impatient members of society to receive a large share of consumption at the beginning, which then falls over time. More patient individuals will therefore have a

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<sup>21</sup> See Arrow (1950).

<sup>22</sup> There is plenty of empirical evidence supporting the claim that different people have different rates of time preference. Warner and Pleeter (2001), for instance, found that individual discount rates can vary between 0% and 30%.

<sup>23</sup> Decreasing absolute risk aversion is an entirely plausible assumption given that the share of wealth invested in risky assets increases with income in both developed and developing countries (Ogaki and Zhang, 2000).

higher rate of consumption growth. Under plausible assumptions,<sup>24</sup> the tolerance to fluctuations of patient individuals increases over time, relative to impatient individuals.

In summary: (1) the weights on discount rates are proportional to tolerance to fluctuations; and (2) the tolerance of patient individuals, and thus the weight placed on their discount rate, increases over time. It follows that the collective *utility* discount rate decreases with time.

This result is only relevant to climate change policy if it is believed that the utility discount rate for social cost benefit analysis should reflect an aggregation of individual preferences. Over relatively short time horizons, this may be sensible. Over longer time horizons, for the reasons discussed in section 2 above, social parameters should not be based upon revealed individual impatience.

#### **4.4 Time inconsistency**

Employing a declining *utility* discount rate can give rise to problems of time inconsistency (Strotz, 1956).<sup>25</sup> Time inconsistency (or ‘dynamic inconsistency’) arises when a plan determined to be optimal at one date is no longer optimal when considered at a later date. In other words, the optimal plan depends upon the evaluation date. As such, unless a planner can commit future planners to the original plan, it will eventually be abandoned. Solow (1999) comments that this ‘sounds like a poor way to run a railroad.’<sup>26</sup> Note that the problem of time inconsistency arises from time-varying utility

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<sup>24</sup> Namely the assumption of increasing absolute tolerance to consumption fluctuations. This means that people with higher level of consumption are more tolerant to fluctuations.

<sup>25</sup> Heal (1998) proves that almost all types of declining utility discount rates generate time inconsistency problems.

<sup>26</sup> Hyperbolic discounting has been so successful precisely because this time inconsistency allows it to explain phenomena such as procrastination and addiction, where well-being is not maximised.

discount rates – it does not arise for time-varying *consumption* discount rates when the underlying *utility* discount rate is constant.

Faced with potential time inconsistency, a government without a commitment mechanism can formulate policy in a ‘naïve’ or ‘sophisticated’ manner. Neither situation is satisfactory. The sophisticated government takes into account the fact that future governments will have an incentive to deviate from its optimal (committed) policy. The situation may be modelled as an intertemporal game played with its successors. The government makes policy as the best response to successive government’s best responses, retaining credibility and, as Barro (1999) and Karp (2005) illustrate, time-consistency.<sup>27</sup> The result, however, is not Pareto optimal. In contrast, the ‘naïve’ government presses ahead with dynamically inconsistent policy, ignoring the fact that future governments will find its policies to be sub-optimal. This is also clearly sub-optimal.<sup>28</sup> From the perspective of the current ‘naïve’ government, its optimal policy will not be adhered to.

Despite these results, several commentators do not consider time inconsistency to be a serious problem. Heal (1998) argues that time consistency is a ‘most unnatural requirement’ given that social decisions generally satisfy weaker rationality conditions than individuals do. Henderson and Bateman (1995) present a similar view. Spackman (2002) states that ‘it is hard to see any serious philosophical or policy objection to [time

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<sup>27</sup> Interestingly, under certain conditions discussed by Barro (1999) this Nash equilibrium policy ends up being equivalent to a policy that would have been constructed using a conventional exponentially declining discount rate.

<sup>28</sup> Hepburn (2003), for instance, shows that a naïve government employing a hyperbolic *utility* discount rate in the management of a renewable resource can unwittingly manage the resource into extinction.

inconsistency], if it reflects the considered preferences of people at the time that each decision is made.'

More importantly, it is worth reemphasising that the problem only arises when the *utility* discount rate is time-varying.<sup>29</sup> In contrast, incorporating uncertainty in economic growth rates, which is a more substantial issue, generates declining *consumption* discount rates. When the *consumption* discount rate is declining, although policy decisions are likely to be suboptimal *ex post*, this does not make them time-inconsistent. Newell and Pizer (2003) remind us that in an uncertain world, decisions that are sensible *ex ante* often turn out to be regrettable *ex post*.<sup>30</sup>

Finally, even if time inconsistency problems are produced by declining discount rates, as a practical matter it seems likely that such problems are likely to be substantially less worrying than policy reversals prompted by political or external shocks.

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<sup>29</sup> And even with declining *utility* discount rates, one might speculate, given the results in Gollier and Zeckhauser (2005), that there could be a way around the problem. There, although the collective policy shows a declining discount rate, it is not time-inconsistent. This is because each individual agent, with a constant utility discount rate, adopts a time-consistent consumption plan. As such, even though the social planner may be inclined to adjust the plan as time passes, each individual agent has no incentive to do so. So, in theory, decentralisation may effectively create a commitment mechanism that resolves the time inconsistency problem. How this would be implemented in practice is unclear. Perhaps different (but constant)  $\delta$  could be employed for projects of different lengths?

<sup>30</sup> Further discussion of time inconsistency is provided by Hepburn and Groom (2006).

## 5. Fairness between generations

### 5.1 Dynamic efficiency and equity

The previous section showed that under uncertainty, declining social discount rates are likely to be necessary for efficiency. While this also increases the weight placed on the future, as compared with constant discounting at the initial rate, it by no means guarantees an equitable intergenerational allocation.<sup>31</sup> As in the static case, there is no particular reason to assume that a dynamically efficient allocation is also equitable.<sup>32</sup>

For long-term problems, some analysts argue that equity should come first. For instance, Howarth (2003) argues that the moral duty to ensure that opportunities are sustained from generation to generation should override considerations of efficiency. Page (1997) similarly argues that we have a duty — analogous to a constitutional requirement — to ensure intergenerational equity first, before efficiency is considered.<sup>33</sup> Such considerations have prompted a range of alternative approaches to intergenerational trade-offs.

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<sup>31</sup> Interestingly, however, under some specifications declining discount rates are necessary for intergenerational equity. See Chichilnisky (1996, 1997), Li and Löfgren (2000) and Heal (2003).

<sup>32</sup> For instance, when production depends on capital, labour and an exhaustible natural resource, maximising net present value at any positive discount rate will eventually force consumption to zero — arguably an unfair outcome for future generations — even though non-decreasing consumption is feasible (Asheim and Buchholtz, 2003).

<sup>33</sup> Pigou (1932) agreed that such duties existed, describing the government as the ‘trustee for unborn generations’. But Schwartz (1978) and Parfit (1983) question whether the notion of a duty to posterity is well-defined, on the grounds that decisions today not only determine the welfare but also the identities of future humans.

## 5.2 Alternative approaches<sup>34</sup>

Schelling (1995) provides a particularly insightful critique of conventional discounting in the climate change context. He points out that employing a pure rate of time preference for long-term problems is inappropriate because it is based upon the impatience of individuals with respect to their *own* consumption. He argues instead that investments for people in the distant future should be considered much like foreign aid. For instance, investment now to reduce future greenhouse gas emissions should not be viewed as saving, but rather as a transfer of consumption from ourselves to people living in the distant future, which is similar to making sacrifices now for the benefit of our contemporaries who are distant from us geographically or culturally.<sup>35</sup> The only difference is that the transfer mechanism is no longer the ‘leaky bucket’ of Okun (1975), but rather an ‘incubation bucket’, where the gift multiplies in transit.

Schelling’s alternative — the ‘utility function approach’ — would drop the use of a discount rate, and instead present policy makers with a menu of climate change mitigation investments along with their impact on future consumption in each world region (and time period) for each investment. The debate would then focus on the appropriate utility function to employ to value consumption increases in different regions at different times. His approach has the merit of insisting on transparency in the weights placed on consumption flows at each point in time and space, and this is to be welcomed. However, the questions that must be answered to apply this approach are essentially the same as those needed for conventional discounted cost benefit analysis. For instance, what utility function should underpin the weights? Should the weights

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<sup>34</sup> This section is adapted from Hepburn (2006).

<sup>35</sup> The analogy to foreign aid, while extremely helpful, highlights an important missing element of the problem, discussed in footnote 1. Many people are concerned to prevent the loss of species and ecosystems because of their intrinsic value, not merely because of their value to humans living in the future.

reflect the human tendency to discount for unfamiliarity along temporal, spatial and cultural dimensions? Indeed, Schelling's approach appears to be consistent with conventional discounted cost-benefit analysis with  $\delta=0$ , and where the stress is upon the disaggregated information for each region at each moment in time.

Kopp and Portney (1999) suggest a proposal going one step further. A detailed description of the likely effects — across time and space — of a policy being implemented or not would be presented to a random sample of the population, who would vote on the policy. By varying the estimate of the costs for different respondents, a willingness to pay locus for the policy would be determined. Their approach has the appeal of valuing the future by asking citizens directly, rather than examining their behaviour or by reference to particular moral judgments to determine a discount rate.<sup>36</sup> Problems with this approach, as Kopp and Portney (1999) note, include the usual possible biases in such stated preference surveys and the difficulty of providing adequate information for an appropriate decision on such a complex topic. An additional feature is that the interests of future generations are reliant upon current voters incorporating them into their preferences.

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<sup>36</sup> Page (2003) also proposes that voting should be considered as an alternative to discounted cash flow analysis for important long-term public decisions, arguing it is more likely to produce fair outcomes.

## 6. Conclusions

Given the sheer scale of the challenge, finding the most efficient feasible response to climate change is critical. Determining the efficient level of climate change mitigation (and adaptation) requires an assessment — in one way or another — of the shadow price of consumption in the future relative to consumption today. Climate change also raises difficult issues of intergenerational equity, which are arguably as important as efficiency. Certainty, international collective action will be impossible to achieve if equity issues are ignored. Assessing efficiency and equity properly is impossible without a forecast of future growth rates (and hence future consumption levels) that explicitly accounts for uncertainty and heterogeneity between (if not also within) countries.

For policy purposes, this information must then be aggregated over states of nature, space and time. All the decision-making approaches discussed above implicitly or explicitly provide an aggregation mechanism, which inevitably involves some ethical issues.

Finally, we draw five specific conclusions about the discounting of climate change damages:

1. Future climate policy may involve action on such a large scale as to shift our economic path in a non-marginal fashion. Under such circumstances, conventional discounted cost-benefit analysis is inapplicable. Instead, analysis should proceed using a utility discount rate to compare the stream of social welfare with and without the climate policy.
2. The utility discount rate,  $\delta$ , used for social decision-making should not be estimated based upon revealed individual impatience, but should reflect the risk of societal collapse. On this basis, the appropriate utility discount rate is smaller

than the current HM Treasury rate of 1.5%, and although it is positive,  $\delta$  is probably below 0.5% and possibly 0% to a first approximation.

3. Probabilistic forecasts of future growth rates should be used to determine certainty-equivalent shadow consumption discount rates. These will decline with time in the long run. They will also be different for different countries.
4. No specific schedule of declining discount rates for the UK is recommended here, because this should be based upon estimates of the distribution of future growth rates. Nevertheless, point 2 above suggests that the resulting schedule is likely to be lower from  $t=0$  onwards than current HM Treasury guidance.
5. Concerns about time inconsistency are relatively minor, and only arise with a declining *utility* discount rate,  $\delta$ . (Point 2 suggests  $\delta$  is both very small and constant.)

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## Appendix

This short note proves that utility is increased when a social planner makes an investment — represented by cash flows  $\Delta(t)$  — which has a positive net present value. The proof holds provided the investment is marginal. Let  $u : \mathfrak{R}^+ \rightarrow \mathfrak{R}$  be a utility function, let  $\delta(t)$  denote a (time-variable) utility discount rate, so that the corresponding consumption discount rate is:

$$\rho(t) = \delta(t) - \frac{d}{dt} \log u'(c(t)) \quad (1)$$

**Proposition 1.**

$$\int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} \Delta(t) dt > 0 \quad (2)$$

$$\implies \int_0^\infty u(c(t) + \Delta(t)) e^{-\int_0^t \delta(\tau) d\tau} dt > \int_0^\infty u(c(t)) e^{-\int_0^t \delta(\tau) d\tau} dt \quad (3)$$

**Proof.** Substitute equation (1) into the left hand side of equation (2):

$$\int_0^\infty e^{-\int_0^t \rho(\tau) d\tau} \Delta(t) dt = \int_0^\infty e^{-\int_0^t \delta(\tau) d\tau} \exp\left(\int_0^t \frac{d}{d\tau} \log u'(c(\tau)) d\tau\right) \Delta(t) dt \quad (4)$$

$$= \int_0^\infty e^{-\int_0^t \delta(\tau) d\tau} \exp\left(\log \frac{u'(c(t))}{u'(c(0))}\right) \Delta(t) dt \quad (5)$$

$$= \frac{1}{u'(c(0))} \int_0^\infty e^{-\int_0^t \delta(\tau) d\tau} u'(c(t)) \Delta(t) dt \quad (6)$$

Now for a marginal investment,  $\Delta(t)$  small, the following Taylor approximation holds:

$$u(c(t) + \Delta(t)) \approx u(c(t)) + \Delta(t) u'(c(t)) \quad (7)$$

Substitute equation (7) into equation (6) and then into equation (2):

$$\int_0^\infty e^{-\int_0^t \delta(\tau) d\tau} [u(c(t) + \Delta(t)) - u(c(t))] dt > 0 \quad (8)$$

From which it follows that equation (3) holds. ■