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Declining Discount Rates: The Long and the Short of it

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Abstract. The last few years have witnessed important advances in our understanding of time preference and social discounting. In particular, several rationales for the use of time-varying social discount rates have emerged. These rationales range from the *ad hoc* to the formal, with some founded solely in economic theory while others reflect principles of intergenerational equity. While these advances are to be applauded, the practitioner is left with a confusing array of rationales and the sense that almost any discount rate can be justified. This paper draws together these different strands and provides a critical review of past and present contributions to this literature. In addition to this we highlight some of the problems with employing DDRs in the decision-making process, the most pressing of which may be time inconsistency. We clarify their practical implications, and potential pitfalls, of the more credible rationales and argue that some approaches popular in environmental economics literature are ill-conceived. Finally, we illustrate the impact of different approaches by examining global warming and nuclear power investment. This includes an application and extension of Newell and Pizer ['Discounting the benefits of climate change mitigation: how much do uncertain rates increase valuations?' *Journal of Environmental Economics and Management* **46** (2003) 52] to UK interest rate data.

Key words: global warming, intergenerational equity, social cost benefit analysis, time inconsistency, uncertainty, time varying discount rates

1. Introduction

Debates about discounting have always occupied an important place in environmental policy and economics. Like all other investments, investment in the environment involves incurring costs today for benefits in the future. Whether a public investment is efficient or not is determined by social cost benefit analysis (CBA). Where welfare is Utilitarian, the socially efficient level of investment is attained by investing in projects where the net present value

(NPV), determined by discounting costs and benefits at the social discount rate (SDR) over the time horizon, is greater than zero.[1] It follows that the level of the SDR is critical in determining whether an individual public investment or policy will pass a CBA test.

Quite separately from arguments over whether the discount rate should be positive or not (e.g., Olson and Bailey 1981; Broome 1992), economists and others have argued at length over which of several potential discount rates should be used as the SDR (e.g., Marglin 1963; Baumol 1968; Lind 1982). Several candidates exist, the most widely recognised of which are the social rate of return on investment (r) and the rate at which society values consumption at different points of time, the Social Rate of Time Preference (δ). The distinction between these discount rates is most important in the second best world in which distortions to the economy, such as corporate and personal taxes or environmental externalities, prevent these rates from being equalised. The choice of SDR is inherently complicated in such situations and is dependent upon a wide variety of factors. These factors include: the extent to which public investment displaces or generates consumption or private investment throughout the lifetime of the project, the extent to which project risk is captured by the discount rate, and assumptions concerning reinvestment (Lind 1982; Portney and Weyant 1999). However, one thing common to much of the past literature is that, whatever the rate chosen, the relative weights applied to all adjacent time periods would be invariant across the time horizon considered. That is, discounting would be exponential.

A common criticism of discounting is that it militates against solutions to long-run environmental problems: for example, climate change, biodiversity loss and nuclear waste, which need to be evaluated over a time horizon of several hundred years. The question arises: What is the appropriate procedure for such long time horizons? There is wide agreement that discounting at a constant positive rate in these circumstances is problematic, irrespective of the particular discount rate employed. With a constant rate, the costs and benefits accruing to generations in the distant future appear relatively unimportant in present value terms. Hence, decisions made today on the basis of CBA appear to tyrannise future generations and in extreme cases leave them exposed to potentially catastrophic consequences. Such risks can either result from current actions, where future costs carry no weight, e.g. nuclear decommission, or from current inaction, where the future benefits carry no weight, e.g. climate change. The intergenerational issues associated with discounting have puzzled generations of economists. Pigou (1932) referred to the deleterious effects of exponential discounting on future welfare as a ‘defective telescopic faculty’. More recently Weitzman (1998) summarises this puzzle succinctly when he states:

'to think about the distant future in terms of standard discounting is to have an uneasy intuitive feeling that something is wrong, somewhere'.

Discounting also appears to be contrary to the widely supported goal of 'sustainability', which by most definitions implies that policies and investments should contribute to securing sustained increases in per capita welfare for future generations (Atkinson et al. 1997). Also, by attaching little weight to future welfare conventional discounting appears to ignore any notion of intergenerational equity.

A recently proposed solution to this problem is to use a discount rate, which declines with time, according to some predetermined trajectory, thus raising the weight attached to the welfare of future generations. It is possible that using a declining discount rate (DDR) could make an important contribution towards meeting the goal of sustainable development.

So, what formal justifications exist for using a DDR and what is the optimal trajectory of the decline? This paper reviews recent contributions addressing these two issues in different ways. We tie together the different approaches – some deterministic, others based on uncertainty, some based upon intergenerational equity, others on considerations of efficiency – and in so doing we highlight some important theoretical and practical issues that arise with DDRs.

The paper proceeds as follows. Section 2 provides a brief review of the theory underpinning social discount rates. Section 3 presents the arguments for DDRs in a deterministic world. In Section 4, we review the literature on uncertainty and discount rates and show that the argument for DDRs is most compelling here. In Section 5, we examine the arguments for DDRs founded on intergenerational equity and in Section 6, we summarise some of the hyperbolic discounting literature. Practical issues arising from the use of DDRs in policy making are considered in Section 7 and two case studies are examined in Section 8.

2. Social Discount Rates: A Brief Review

2.1. THE RAMSEY MODEL

In this section, we take the Ramsey growth model as our starting point and describe the derivation of the socially optimal discount rate. In so doing we provide the general framework in which the ensuing discussion of DDRs takes place and show the relationship between the social rate of time preference δ, the private return to investment, i, the social rate of return to investment, r, and the 'utility discount rate' or rate of pure time preference, ρ. Each of these rates is a contender for use as the SDR, where the appropriate discount rate for use in CBA depends upon the numeraire employed. For example, the utility discount rate, ρ, is the appropriate discount rate for costs and benefits that are measured in utility. Alternatively, in the Ramsey model r, i, and δ represent the appropriate SDRs when costs and benefits are measured in consumption equivalents, as is usual practice in CBA. In both cases, the SDR represents the rate of change of the value or shadow price of the numeraire (Dasgupta 2001).[2]

The conventional approach to CBA is based on neoclassical growth theory and underpinned by utilitarian ethics. Where the numeraire is units of consumption, the SDR is endogenously determined within the Ramsey framework by optimal saving, consumption and production decisions over time. Although there are a number of abstractions in this model, which often exist for the sake of tractability, it represents a useful starting point for the discussion of the discount rate and its economic and ethical content. Ultimately, we present the Ramsey model as a normative approach to the social discount rate.[3]

In the Ramsey model social welfare is represented by the intertemporal sum of the utility of a representative agent. Social welfare is maximised over an infinite time horizon, discounted at the utility discount rate, ρ.[4] Intertemporal welfare is assumed to be time-separable and the continous time maximand for the representative agent is therefore:

$$U(c(t)) = \int_0^{\infty} u[c(t)]\exp(-\rho t)\mathrm{d}t \tag{1}$$

where the felicity function, u (.), is time invariant and has the following properties: $u'(.) > 0, u''(.) \leq 0$.[5] Welfare is maximised in an economy where capital, $k(t)$, yields output, $f(k(t))$, which can be devoted to consumption or investment subject to the intertemporal constraint:[6]

$$\dot{k}(t) = f(k(t)) - c(t) \tag{2}$$

Maximising (1) subject to (2) yields the Euler equation:

$$u'(c(t))f'(k(t)) + u''c(t))\dot{c}(t) - \rho u'(c(t)) = 0 \tag{3}$$

which when simplified yields the familiar Ramsey rule:

$$r = \rho + \theta g = \delta \tag{4}$$

where $r = f'(k(t))$ is the social marginal productivity of capital, $g = \dot{c}(t)/c(t)$, and θ represents preferences for smoothing consumption over time and is known as the elasticity of inter-temporal substitution. θ is a measure of the curvature of the utility function and is mathematically equivalent to the coefficient of relative risk aversion: $\theta = -\frac{u''}{u'}c$.

In the absence of externalities and other distortions, the social and private (i) rates of return to capital coincide: $r = i$.[7] The term δ is defined as the social rate of time preference, which reflects the change in relative value that society places on units of consumption at adjacent periods of time. δ can also be thought of as the rate of return to consumption and as a consequence it is frequently referred to as the Consumption Rate of Interest (CRI). In general, δ and CRI are considered to be conceptually different, the former representing the intertemporal weights placed on consumption by society, and the latter representing the same but for individuals. It is frequently the case that the latter is used to measure the former using observed rates of return on savings (Lind 1982).

In sum, Equation (4) shows that on the optimal path the social planner will choose consumption and savings such that the social rate of time preference (consumption rate of interest) is equal to the marginal productivity of capital. In the competitive economy without distortions, social and private rates of return coincide.

2.2. INTERPRETATION AND EXTENSIONS

Ramsey interpreted equation (1) as the maximand of an infinitely lived representative agent acting as a trustee for current and future generations in choosing consumption and saving. Central to this interpretation is a bequest motive: the infinitely lived agent reflects an immortal extended family containing many finitely lived altruistic families. These families are connected by a series of intergenerational transfers to their children who in turn give to their children, etc. Although there has been criticism of this approach, there is at least some agreement that this abstraction represents a convenient framework for long-term analysis (Manne 1995; Stephane et al. 1997; Tóth 2000).[8]

One deficiency from the perspective of environmental economics is the absence of explicit consideration of stocks and flows of environmental assets. This deficiency has been addressed in numerous papers in the realm of optimal growth in which stocks of environmental resources ($s(t)$) are explicitly introduced as a determinant of utility in order to represent amenity values and other preferences for the environment (see e.g. Brock 1977, Chichilnisky 1997, Heal 1998, Li and Löfgren 2000). In such cases instantaneous utility is represented by $u(c(t), s(t))$, and the behaviour of environmental stocks captured by associated equations of motion reflecting the extent to which the resource is renewable and the impact of consumption or production on the environment. Such analyses are frequently directed to the question of optimal and sustainable growth and are also explicitly concerned with notions of intergenerational equity. In effect, such approaches extend the realm of preferences that count in CBA to more explicitly include those of future generations.

Ramsey (1928) described the discounting of utility, that is, placing different weights upon the utility of different generations, as 'ethically indefensible'. Harrod (1948) famously stated that discounting utility represented 'rapacity and the conquest of reason by passion.'[9] Ramsey reflected this belief in his normative analysis of optimal growth by assuming that $\rho = 0$, and since that time these opinions have been the subject of much contemplation by economists and philosophers alike.[10] However, given (3) it should be clear that this by no means implies that costs and benefits measured in units of consumption should not be discounted. With positive growth and concave utility the SDR will be positive, hence discounting consumption streams in CBA can be synonymous with the equal treatment of generations' welfare (Lind 1995).

It is common in theoretical work to employ a positive, time invariant utility discount rate reflecting both alternative beliefs about time preference and the need for tractability, particularly in defining optimal paths. This practice is not without some theoretical basis. Olson and Bailey (1981) look at the implications of assuming $\rho = 0$ for optimal consumption paths. One finding is that the high levels of saving implied by this assumption do not tally well with the empirical evidence. In short, assuming that $\rho = 0$ entails 'excessive sacrifice' by, that is, immiseration of current generations for the sake of the future. This, they argue, provides a strong rationale for assuming a positive rate of time preference. Asheim et al. (2001) have recently demonstrated that the assumption of zero utility discounting (or 'equity', as they call it) does not rule out the existence of an optimum when technology under certain reasonable technologies. Furthermore, Asheim and Buchholz (2003) show that the 'excessive sacrifice' argument is circumvented under plausible technologies and where utility is more concave.[11]

In a more general preference framework, Koopmans (1960) took an axiomatic approach to the question of individual utility discounting or impatience. He showed that the existence of impatience, that is, the use of a positive utility discount rate, which is constant over time $(\rho > 0, \rho_t = \rho)$ is implied by the presence of a number of very particular axioms of rationality concerning the intertemporal welfare function $U(c)$, among other things. For example, $U(c)$ must be continuous in its arguments, stationary over time and satisfy a condition known as 'period independence'. That is, preferences for benefits and costs at a particular period of time are independent of those in the past or future. Indeed, Koopmans went on to show that the same axioms imply that $U(c)$ takes the time-separable form shown in (1). Of course, like any separability assumption, time-separable preferences are not entirely defensible and are considered by many to be problematic. Barro and King (1984) for example, show how time-separability places restrictions upon the relative responses of consumption and leisure to changes in relative prices and permanent income. Intuitively, it seems unlikely that current and future *tastes* would be independent of decisions made in the past.

Discounting utility at a constant rate, ρ, insures that the decisions made by the representative agent/social planner are time consistent, that is, the planner will not change his plan purely as a result of the passage of time (Heal 1998; Gollier 2002a). Indeed, time consistency is implied by Koopman's period independence assumption. One feature of discount rates that vary over time, on the other hand, is that they tend to invoke time inconsistent behaviour and all its associated travails (Strotz 1956; Barro 1999; Hepburn 2003). In this regard, in addition to the theoretical and ethical arguments concerning the utility discount rate, a great deal of attention has been paid to the discount rates that individuals actually employ. The so-called hyperbolic discounting literature provides considerable evidence that individuals use time varying discount rates in their everyday decision making (e.g., Loewenstein and Prelec 1992; Henderson and Bateman 1995; Frederick et al. 2002). It is frequently posited that such time preferences can explain behaviour as diverse as 'savouring' or 'dread' effects and seemingly irrational behaviour such as addiction and other 'slippery slope' phenomena. Weitzman (1998) notes that, although this behaviour typically refers to short-run behaviour there is an evolutionary argument for using time varying discount rate for the longer term: since we only observe those who survive, hyperbolic discounting must be an effective survival strategy. Similar ideas are developed by Dasgupta and Maskin (2002). Such observations generate something of a puzzle when one considers the ethical underpinning of CBA described above, that is, that preferences count. The evidence raises the question: Is a model of time preferences that describes irrational and often inefficient behaviour a suitable model for social CBA? This is an issue that is discussed further in Section 7.

The discussion in the previous sections has shown that to a great extent the social discount rate can be considered to be a derived concept. In the Ramsey world the SDR emerges from the optimising economy, while, Koopmans focussed the discussion about the utility discount rate on the underlying axioms of individual rationality that one is prepared to adhere to. In this sense the SDR is not 'ethical raw material' (Dasgupta 2001).

2.3. SELECTION OF THE DISCOUNT RATE

The Ramsey rule in equation (3) shows why it is valid to consider the social rate of time preference, δ, and the rate of return on capital, hereafter r, as candidates for the socially efficient discount rate for projects or policies whose costs and benefits are measured in consumption equivalents. If projects are to be financed by current consumption or investment, then the rate of return on these projects ought to be compared to that which prevails on the optimal path. Marginal investment will be efficient if projects are selected in this way. In the perfectly competitive paradigm, all rates are equal and hence it does not matter which rate: i, r, or δ, is used for CBA.

Furthermore, true to the ethical underpinning of CBA described above, the Ramsey rule reflects the particular facets of individual preferences that provide the rationale for discounting the future in a deterministic world: (1) *impatience*, reflected by the utility discount rate or pure rate of time preference, ρ, and (2) *the wealth effect* represented by the term θg, where θ is greater than zero if households are averse to consumption fluctuations.[12] The wealth effect describes how the representative agent will place less value upon additional units of consumption in the future if her belief is that incomes at that time will be higher as a result of economic growth. This effect will be amplified if there is a strong desire to smooth consumption over time. As we shall see in Section 4, when uncertainty with regard to growth is introduced, preferences for risk also play a role in determining the socially efficient discount rate, as reflected by the coefficient of relative risk aversion: θ.

In general the Ramsey model will not be descriptive of the economy. In reality, it will not be true that $i = r = \delta$ and the debate about discounting has concerned when and whether it is appropriate to use i, r or δ, or some combination thereof (Baumol 1968; Lind 1982). For example, assuming for the moment that $i = r$, distortionary income and corporation or profit taxes will in general cause the rate of return on capital to differ from the social rate of time preference. In general it will be the case that $r > \delta$.[13] Imperfect competition and externalities in production will cause private and social rates of return to capital to diverge: $i \neq r$. Furthermore, the appropriate discount rate for a particular project will depend upon the extent to which a project is funded by consumption or by displaced private investment. It has been argued that, other things equal, a project funded entirely by consumption should be discounted by δ while a project funded entirely by investment should be discounted by the private rate of return on capital, i (Lind 1982). Subsequently, others have suggested that projects funded by a mixture of the two should be discounted at a rate which reflects an average of the two rates, weighted by the proportions in which consumption and private investment finance the project (Haveman 1969). Another suggestion is to convert all costs and benefits into consumption equivalents using the *shadow price of capital* approach and then to use the δ as the SDR (see e.g. Bradford 1999).[14] In addition to these factors, the rate of reinvestment of returns and the riskiness of private versus public investments are also considered to be determinants of the socially efficient discount rate (Baumol 1968). With regard to risk it is commonly thought that the risk free rate of return is appropriate for the appraisal of public projects due to risk pooling available to governments (Samuelson 1965; Arrow 1966; Lind 1982). Nevertheless, each of these factors requires consideration when determining the correct level of the discount rate in what Baumol called the ‘dark jungles of the second best’ (Baumol 1968).

2.4. COMMON PRACTICE AND DEFINITIONS

There are many factors that need to be considered when determining the socially efficient discount rate for use in CBA. This leads to the difficult prospect of different discount rates for different projects (Lind 1982). It is common practice for governments to abstract from detailed adjustments, such as those described above, and employ more practical rules of thumb. For example, in the ‘Green Book: Appraisal and Analysis in Central Government’, the UK government recommends the use of the social rate of time preference as the test discount rate for CBA (HM Treasury 2003). This rate is recommended for use across all departments, for all projects and is calculated to be $\delta = 3.5\%$.[15] The policy in the US is more tailored. It is proposed that investments external to the government are evaluated at a rate reflecting the average return in the private sector, currently 7%. Alternatively, internal investments are evaluated at the rate of return on treasury bonds, 4%. The shadow price approach outlined above is also suggested for certain appraisals where the social rate of time preference is assumed to be reflected by the return on treasury bonds (Newell and Pizer 2001). The use of the rate of return on treasury bonds reflects the commonly held view that it is the risk free rate of return that is applicable to public investments. In any event, whatever the choice of SDR, the usual practice is to employ the current estimate for all periods of time. One of the few exceptions to this rule is the UK government, which has recently introduced a declining schedule of discount rates to the Green Book for use in long-term projects (HM Treasury 2003).[16]

Projects are appraised by establishing their Net Present Value (NPV) determined by summing up the net benefits that occur at each moment in time, where the net benefits are determined using accounting prices and are weighted by the *discount factor*;

$$a(t) = \exp\left(\int_0^t -\delta(s)\mathrm{d}s\right) \tag{5}$$

which, reflects the value of the numeraire in each time period. As stated above, the discount *rate* is most frequently defined as the rate of change of the value of the numeraire or discount factor. However, there is a distinction to be made between average and marginal rates. The average rate of discount, $\delta_a(t)$, can be thought of as the rate, which if applied constantly for all intervening years would yield the discount factor, $a(t)$. It is therefore defined as follows:

$$\exp(-\delta_{\mathrm{a}} t) = a(t) \tag{6}$$

The average discount rate can be derived for any time period from the simple rearrangement of (6):

$$\delta_{\mathrm{a}}(t) = -\frac{1}{t}\ln a(t) \tag{7}$$

The marginal rate of discount is the rate of change of the discount factor and can be calculated as follows:

$$\delta_{\mathrm{m}}(t) = -\frac{\frac{\partial}{\partial t}a(t)}{a(t)} = -\frac{\dot{a}(t)}{a(t)} \tag{8}$$

It is common practice to assess the NPV of a project using a constant discount rate. Frequently, this amounts to assuming an exogenous discount rate and hence a partial equilibrium framework on the basis that the project is too small to influence the economy as a whole. Clearly, where the discount rate is constant for all time periods, marginal and average rates coincide: $\delta_{\mathrm{a}} = \delta_{\mathrm{m}} = \delta$. However, where the discount rate is time dependent, for example, where we have declining discount rates (DDRs), this distinction can become important.

3. Declining Discount Rates in a Deterministic World

3.1. GROWTH (g) AND CONSUMPTION SMOOTHING (θ)

The Ramsey rule in equation (4) shows the determinants of the socially efficient equilibrium discount rate: pure impatience, ρ, the desire for consumption smoothing, θ, and growth, g. With certain knowledge of each of the parameters on the RHS of (4) the social rate of time preference, δ, is known with certainty, and in the competitive economy we know that it will be equal to the private and social return on capital. Given these consumption based determinants of δ, it is interesting to consider its level and how it might change over time.

Firstly, as Dasgupta (2001) makes clear, given the definition of δ in (4), negative growth could produce a negative social discount rate.[17] Similarly, if θ and ρ are constant, and growth is known to be declining then the socially efficient discount rate will be declining. Formally, assuming ρ is constant, it follows from the definition of δ:[18]

$$\frac{\partial}{\partial t}\delta(t) = \frac{\partial \theta}{\partial t}g + \theta\frac{\partial g}{\partial t} \tag{9}$$

Hence, there are a number of situations in which the SDR will be declining in this deterministic setting. Firstly, if we maintain the assumption that $\theta > 0$ and constant over time $\left(\frac{\partial \theta}{\partial t} = 0\right)$ then DDRs will occur if growth is decreasing over time: $\frac{\partial g}{\partial t} < 0$.[19] A diminishing rate of growth is a very real possibility and may be particularly relevant when considering e.g. climate change prevention. With declining growth the appropriate discount rate for the long-term is declining, where $\frac{\partial \theta}{\partial t} = 0$. Clearly, the behaviour of the efficient discount rate over time is highly dependent upon the preferences of the representative household, in particular the level of θ and its evolution over time with changes in income. This is summarised in Proposition 1:

Proposition 1. Assuming that the pure rate of time preference is constant over time, in the deterministic case the socially efficient discount rate will decline unambiguously over time if, for whatever reason, growth is declining (increasing) $\frac{\partial g}{\partial t} < 0\left(\frac{\partial g}{\partial t} > 0\right)$ and preferences are such that $\theta > 0 (\theta < 0)$ and θ is inversely related or unrelated to income, regardless of the level of growth, g.

Proof. The proof comes from inspection of the right hand side of (9). For the second term to be negative requires that where growth is decreasing, $\frac{\partial g}{\partial t} < 0$, (increasing, $\frac{\partial g}{\partial t} > 0$), θ must be positive (negative). For the first term to be non-positive requires that either (i) $\frac{\partial \theta}{\partial t} = 0$, as in the example above; (ii) $g > 0$ and $\frac{\partial \theta}{\partial t} < 0$; or (iii) $g < 0$ and $\frac{\partial \theta}{\partial t} > 0$. Clearly in case (i) if $\frac{\partial g}{\partial t} < 0\left(\frac{\partial g}{\partial t} > 0\right)$ then $\frac{\partial}{\partial t}\delta < 0$ if $\theta > 0 (\theta < 0)$. Case (ii) and (iii) are satisfied if preferences are such that θ is inversely related to income, making the *level* of growth unimportant.

Our first finding is that DDRs can emerge in a deterministic world because of predictable changes in the growth rate and associated changes in preferences for consumption smoothing or risk. Given that θ is mathematically equivalent to Pratt's coefficient of relative risk aversion, the preferences required for each of the cases above can be as follows: Case (i) requires preferences akin to constant relative risk aversion (CRRA), while cases (ii) and (iii) require preferences which are akin to decreasing relative risk aversion (DRRA). Clearly there exists a number of other cases in which DDRs may emerge where the first term and second term are of opposite sign and yet their sum is still negative. In such cases the level of the parameters, θ, g, and their time derivatives are important. We do not isolate these conditions here. It suffices to note that growth, individual preferences and their behaviour over time are important determinants of the SDR in the deterministic case and that fluctuations in the discount rate used for discounting consumption equivalents can be a natural outcome of the traditional Ramsey model. The analysis here also provides a useful introduction to the work of Gollier (2002a, b), which looks at the long-term discount rate under uncertainty, in which case θ reflects risk preferences.

3.2. ENVIRONMENTAL VALUE AND EXTERNALITIES

A second justification for DDRs in a deterministic world arises from the work of Weitzman (1994). It is well known that environmental externalities in consumption or production can cause the social and private rates of return on capital to diverge. Weitzman (1994) provides theoretical conditions for an 'environmental' SDR based upon the social rate of return to capital, which is lower than the private rate. In so doing Weitzman (1994) isolates the conditions under which DDRs emerge. His model incorporates two main ideas: society values environmental resources positively and the production of consumption goods can generate negative environmental externalities. These two basic tenets generate a tension between private investment and public investments in environmental protection, driving a wedge between the private and social rates of return. The model can be thought of as follows. If national income is either consumed, invested or diverted to environmental expenditures we can write:

$$Y(t) = f(k(t)) = C(t) + I(t) + \psi(t)$$

where $f(.)$ is the production technology, $C(t)$ is consumption, $I(t)$ is gross investment and $\psi(t)$ is expenditure on reducing environmental damage, a social cost external to the production process. The relation between environmental expenditures and environmental damage as a proportion of income is defined as:[20]

$$\frac{D}{Y} = G\left(\frac{\psi}{Y}\right) \tag{10}$$

where $G_{\psi} < 0$ and $G_{\psi\psi} > 0$.[21] If investment is increased at time t by a marginal reduction in consumption, keeping environmental expenditures constant, the private rate of return on capital can be thought of as:

$$\frac{\partial Y}{\partial k} = f'(k) \tag{11}$$

Hence, the private rate of return on capital, i, is equal to $f'(k)$. When production generates environmental externalities the social rate of return, r, will differ from the private rate. Rather than modelling the effect of environmental externalities directly, e.g. through explicit modelling of preferences for environmental resources, Weitzman imagines that environmental damage must be maintained at some initial level, $\bar{D}$. Given (10), this can only be achieved by a marginal increase in environmental expenditures, $\psi' = \frac{d\psi}{dY}$, diverted from each unit of incremental output, $\frac{\partial Y}{\partial k}$. Hence, the social rate of return oil investment can be thought of as the rate of return in terms of output minus the rate of increase in expenditure required to maintain environmental standards:

$$r = \frac{\partial Y}{\partial k} - \psi' \frac{\partial Y}{\partial k} = i[1 - \psi'] \tag{12}$$

By taking the total derivative of (10) with respect to Y and solving for ψ' we are left with the term:[22]

$$r = i\left[1 - Z\left(1 + \frac{1}{E}\right)\right] \tag{13}$$

where $Z = \frac{\psi}{Y}$ and $E = -Z\frac{G_\psi}{G}$. The former is the proportion of national income spent on environmental clean-ups and the latter is the elasticity of environmental improvement (i.e. reducing D) with respect to environmental expenditure or the ease with which environmental damage can be reduced.

Notice that the social rate of discount, r, is lower than the private rate, i, for all positive levels of Z and E. For a given level of Z, when the elasticity is low, and environmental expenditures are ineffective at cleaning up environmental damage, this divergence is increased. Weitzman's interpretation, from the perspective of optimal growth, is that this is a signal that the economy is finding prior environmental damage difficult to undo and the solution might be to reduce growth. Alternatively, where the elasticity is high, a better solution might be to increase environmental expenditures (Weitzman 1994).

The implications of this analysis for the discount rate are twofold. Firstly, under fairly general conditions, the existence of consumption externalities reduces the level of the social rate of return below the private rate. This is because society must divide the marginal return from investment between consumption and environmental protection. Secondly, the socially efficient discount rate will be declining over time if the proportion of income spent on environmental goods, Z, is increasing over time. With positive growth this is guaranteed if environmental resources are luxury goods. A similar result holds if the elasticity of environmental improvement is declining over time.

Changing values for the environment were the focus of earlier work on discount rates for environmental projects by Fisher and Krutilla (1975). They suggested that these evolving preferences could be simply captured by assuming that the marginal Willingness to Pay (WTP) or accounting price for the environment would change at some pre-determined rate, say α. WTP would then grow exponentially from some initial level WTP_0 such that $\mathrm{WTP}_t = \mathrm{WTP}_0 \exp(\alpha t)$. The present value of these environmental benefits at time t would then be equivalent to:

$$PV_{\mathrm{WTP}} = \mathrm{WTP}_0 \exp((\alpha - r)t) \tag{14}$$

where r is the SDR, which represents the rate of change in the accounting price for the numeraire. Fisher and Krutilla defined the 'environmental' discount rate as the net rate $\omega = r - \alpha$, suggesting that the change in the accounting price for the numeraire and environmental goods can be captured by this net discount rate. This net rate is constant over time and captures a prediction about the evolution of values from WTP_0.

One example of the mechanism for this process is to assume that the increase in WTP is driven by income growth such that $\alpha = \varepsilon g$, where g is the growth of income and ε is the income elasticity of WTP (Gravelle and Smith 2000). Both Krutilla and Fisher (1975) and Horowitz (2002) reflect on the effect of resource scarcity on WTP for environmental goods in this framework. Both perspectives provide arguments for increasing WTP for environmental goods and hence a reduction in the level of the (time invariant) discount rate for the relevant benefit or cost.[23]

The conditions under which DDRs emerge differ from those of Weitzman (1994). In the Fisher and Krutilla model if the proportion of income spent on environmental goods is increasing, i.e. growth is positive and environmental goods are luxuries $(g > 0, \varepsilon > 1)$, then the environmental discount rate should be lower than r, yet constant over time. DDRs emerge from the Fisher and Krutilla analysis if the parameters which define the evolution of WTP (α) are changing over time.[24] Furthermore, whereas Fisher and Krutilla's discount rate applies solely to environmental costs and benefits, Weitzman's presumably applies to all costs and benefits. The former has become known in the literature as a 'dual discounting' approach, since it refers to discounting different costs and benefits at different rates, and has received considerable attention in climate change modelling (Tol 2003; Yang 2004).

Both Weitzman (1994) and Fisher and Krutilla (1975) have been criticised on a number of counts. In many ways Weitzman's environmental discount rate is difficult to interpret in light of the reduced form set up and, in particular, the absence of an explicit modelling of preferences, environmental goods and externalities. The assumption that some arbitrary environmental standard, $\bar{D}$, must be maintained captures these effects but makes the subtraction of environmental expenditures from the private rate of return in (12) rather ad hoc. More generally, it is thought that deriving the 'effective' or 'environmental' discount rate using (14) or other dual discounting techniques, and using this as the SDR obscures several issues (e.g. Arrow et al. 1995; Horowitz 2002). As can be seen from the discussion of the Ramsey equation (3), there is a completely different set of assumptions that connect the social rate of return to capital, r, growth, g, and preferences (e.g. for the environment, ε). The discount rate is a poor vehicle for capturing these various factors and in the long-term doing so implies a number of very strong structural assumptions. A more widely accepted alternative is to apply the time invariant SDR, e.g. r, to benefits and costs evaluated in consumption equivalents, which reflect the evolution of WTP through time. This disentangles issues of evolving values for the environment from issues of discounting and 'does not change the discount rate to apply to the consumption stream' (Arrow et al. 1995).^{25}

3.3. LIMITATIONS OF THE FINANCIAL MARKETS

One of the fundamental assumptions underlying the use of discounting in cost benefit analysis is that the potential exists for the transfer of resources across generations. That is, the use of the discount rate, e.g. r, to evaluate a project implies that funds could alternatively earn that rate of return in the economy. When considering the long run this implies the existence of a mechanism to facilitate intergenerational transfers of these alternative returns (Lind 1995). There are a number of reasons why this assumption can be called into question. Firstly, financial markets only cover the relative short term, with assets having maturities limited to about 30–40 years. Secondly, although it is possible for investments to be rolled over as and when they mature and there are numerous fiscal and other policies which can redistribute assets across generations (Bradford 1999), it is not clear that governments will be able credibly to commit to such a course of action (Arrow 1966). Some authors suggest that these facts alone provide some further justification for DDRs.

Rabl (1996), for example, effectively interprets ρ in (3) as the aversion to intertemporal fluctuations within a generation and the term θg as the inter-generational inequality aversion parameter. Since financial markets cover only a limited duration, he argues, the duration over which the current generation can redistribute its wealth through time is limited. Hence, ρ should be excluded from estimates of the discount rate for horizons greater than those reflected by the financial markets. He suggests that the SDR should be the social rate of time preference as measured by $\rho + \theta g$ within the duration of financial assets and θg thereafter. This captures the idea that θg represents real growth in the future: real resources for future generations which is not directly constrained by the financial markets. This results in a declining 'stepped' schedule for discount rates. Rabl's interpretation does not represent an attempt to determine the efficient discount rate and is rather ad hoc. It does, however, raise the questions concerning the assumptions underlying discounting in CBA, that is, the existence of intergenerational transfers.

Indeed, it is perhaps the fact that we are uncertain about the long-run market rate of return that the social rate of time preference is frequently used for CBA. In other words, rather than looking to financial markets for answers concerning the correct discount rate for the long-run, perhaps the economic arguments associated with the consumption based determinants of the discount rate will be more fruitful.

4. Declining Discount Rates in an Uncertain world

When uncertainty with regard to the determinants of the discount rate is introduced to the analysis the case for DDRs is even more compelling and much of the recent debate concerning DDRs has centred upon the analysis of uncertainty concerning future states of the world, in particular the social rate of return to capital, r, (e.g. Weitzman 1998), and growth, g (e.g. Gollier 2002a, b, 2004b). In particular, just as Weitzman (1994) introduced preferences for environmental goods as a determinant of the SDR, Gollier shows that in an uncertain world preferences for risk are important.

4.1. UNCERTAINTY ABOUT THE SOCIAL RATE OF RETURN (r)

Weitzman (1998) developed ideas first formalised by Dybvig et al. (1996) and shows how uncertainty regarding the interest rate, r, leads to DDRs.[26] Clearly, there are good reasons to expect that r is uncertain in the long-run. For example, there is uncertainty concerning capital accumulation, the degree of diminishing returns, the state of the environment, the state of international relations, and the level and pace of technological progress. Dybvig et al. (1996) showed that when there is currently uncertainty about the short-term interest rate, the discount rate that should be applied to extremely distant time periods, strictly as $t \rightarrow \infty$, is the lowest rate with a positive probability of being realised. A proof of this is shown in Appendix A. This argument suggests lower socially efficient discount rates at the limit, but says nothing about the path of these rates over time: i.e. the shape of the yield curve.[27] Weitzman (1998) went on to show the relationship between the socially efficient discount rates and the time horizon. He shows that, when agents wish to maximise the expected NPV in choosing between an investment at an uncertain per-period risk free interest rate, $\tilde{r}$, or in a project that yields a sure benefit in period t, the socially efficient discount rate (before the realisation of the uncertain risk free rate) is declining with time. In other words, the yield curve is declining. In order to understand these results we derive Weitzman's *certainty equivalent discount rate*, show a proof that the limit of this discount rate as $t \rightarrow \infty$ is the lowest possible value and provide a numerical example.

With certain discount rates the discount factor is given by $a(t)$ as shown in equation (5) above. When the social rate of return is uncertain however, there are numerous potential states of the world, each with an associated discount factor and probability of realisation. If there are j states in the world then the discount factor at time t associated with each is:

$$a_j(t) = \exp\left(-\int_0^t r_j(s)\mathrm{d}s\right) \tag{15}$$

where it is assumed here that the interest rate can be a function of time: $r(t)$. Given uncertain future discount rates it becomes necessary to derive a summary measure of the discount factor and discount rate. Weitzman uses certainty equivalent analysis for risk-neutral agents and defines the *certainty equivalent discount factor* (CEDF) as the expectation of the discount factor. From this he derives the *certainty equivalent discount rate* (CER).[28] Supposing that each potential discount rate r_j is realised with probability p_j, such that $\sum p_j = 1$ and $r_j \in [r_{\min}, r_{\max}](j = 1, \ldots, n)$. The certainty equivalent discount factor for a risk neutral agent is defined as:[29]

$$A(t) = E\left[\exp\left(-\int_0^t \tilde{r}_j(s)\mathrm{d}s\right)\right] = \sum_j p_j a_j(t) \tag{16}$$

From this it is possible to define both the average and marginal certainty equivalent discount rates at time t, corresponding to the definitions in Section 2: $r_{\mathrm{a}}^{\mathrm{CE}}$ and $r_{\mathrm{m}}^{\mathrm{CE}}$, respectively:

$$\exp(-r_{\mathrm{a}}^{\mathrm{CE}}(t)t) = A(t) \Rightarrow \tag{17}$$

$$r_{\mathrm{a}}^{\mathrm{CE}}(t) = -\frac{1}{t}\ln[A(t)] \tag{18}$$

$$r_{\mathrm{m}}^{\mathrm{CE}}(t) = -\frac{\frac{\partial}{\partial t}A(t)}{A(t)} \tag{19}$$

The former is the rate of discount that if applied in every period from 0 to t would yield the same value as the expected discount factor at time t. The latter is the instantaneous, period-to-period rate.[30] Weitzman (1998) shows that $r_{\mathrm{m}}^{\mathrm{CE}}$ declines continuously and monotonically over time and that its limit as $t \to \infty$ is $r_{\min}$. Moreover, Gollier (2002b) shows that this certainty equivalent rate can be motivated using a no-arbitrage argument; he shows that an arbitrage exists if, prior to realisation of r, (17) does not hold. That is, thinking of the right hand side of (17) as the (uncertain) price of a claim to £ 1 at time t discounted using the certainty equivalent discount factor, and the left hand side as the present value of the benefit, it is clear that in equilibrium both sides must be equal. Hence, the certainty equivalent discount rate is the equilibrium socially efficient rate for risk neutral agents prior to the realisation of $\tilde{r}$.[31]

The mechanics of Weitzman's results are as follows. From (16) and (19) it is easy to show that the certainty equivalent marginal rate can be written as a weighted average of the potential realisations of r:

$$r_{\mathrm{m}}^{\mathrm{CE}} = \sum_j w_j(t) r_j \tag{20}$$

where the weights in this case are simply: $w_j(t) = p_j a_j(t)/\Sigma p_j a_j(t)$ and $\Sigma w_j(t) = 1$. Taking the derivative of this with respect to time we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t} r_{\mathrm{m}}^{\mathrm{CE}} = \sum_j \dot{w}_j(t) r_j = -\sum_j w_j(t)(r_j - r_{\mathrm{m}}^{\mathrm{CE}})^2 \tag{21}$$

which is clearly negative.[32] Finally, $\lim_{t\to\infty} r_{\mathrm{m}}^{\mathrm{CE}} = r_{\min}$ follows from noticing that, where $r_1 = r_{\min}$:

$$\lim_{t\to\infty} \frac{w_j(t)}{w_1(t)} = 0$$

which means that as $t \to \infty$ the weights associated with all but the lowest discount rate tend to zero due to the presence of $a_j(t)$, and yet, since $\Sigma w_j(t) = 1$, the weight for the lowest discount rate, $w_1(t)$, must tends towards 1.[33] The intuition behind this is that since the weights for each realisation $(w_j(t))$ contain the discount factors $a_j(t)$, in scenarios with higher discount rates the discount factors decline more rapidly to zero. As such, the weight placed on scenarios with high discount rates itself declines with time, until the only relevant scenario is that with the lowest conceivable interest rate. In effect, the power of exponential discounting reduces the importance of future scenarios with high discount rates to zero, since the discount factor in these scenarios more rapidly approaches zero. Since in the ex ante equilibrium the certainty equivalent rate of discount must equal the socially efficient discount rate in all periods of time, this results in a SDR, which declines over time.

4.1.1. *Numerical example of Weitzman's CER.* Appendix B works through an explicit example of Weitzman's certainty equivalent discount rate. Table I shows the resulting schedule of marginal and average discount rates over continuous time assuming that $(r_1, r_2) = (5\%, 2\%)$ and $(p_1, p_2) = (0.5, 0.5)$. Table I reflects the aspects of the certainty equivalent discount rate described above. Both the average and the marginal certainty equivalent rates are declining monotonically through time while approaching the lowest possible realisation in the long-run: $r_{\min} = 2\%$.

4.2. THE NEED FOR AN ANALYSIS OF PREFERENCES

Weitzman's argument seems quite convincing: uncertainty in the discount rate itself leads to an arbitrage in which the socially efficient discount rate is a declining function of time. In addition, the apparent ease of application

Table I. Numerical example of Weitzman's certainty equivalent rate

	Year (t)				
	10	50	100	200	500
Discount factor ($a_1(t)$)	0.819	0.368	0.135	0.018	0.000
Discount factor ($a_2(t)$)	0.607	0.082	0.007	0.000	0.000
CEDF ($A(t)$)	0.713	0.225	0.071	0.009	0.000
Marginal CE ($r_{\mathrm{m}}^{\mathrm{CE}}$)	3.277%	2.547%	2.142%	2.007%	2.000%
Average CE ($r_{\mathrm{a}}^{\mathrm{CE}}$)	3.388%	2.983%	2.645%	2.345%	2.139%

renders it appealing to the practitioner (see Appendix B). However, Gollier (2004a) argues that Weitzman's logic relies critically upon a tacit assumption that the current generation should bear the risk of variation in the SDR. He illustrates this point by using the opposite assumption.

Weitzman's certainty equivalent rate defines the discount rate that should be used when the objective is to maximise the Expected Net Present Value (ENPV) of investments given uncertainty in the interest rate. For example, an agent may wish to compare the return to an investment of £ 1 with fixed future benefit, say £ Z in year T, to an alternative investment with a random rate of return, $\tilde{r}$. She ranks these alternatives by calculating the ENPV. Following Gollier (2004a) in such a case a project is efficient under the ENPV rule if:

$$\mathrm{ENPV} : ZE[\exp(-\tilde{r}T)] - 1 \geq 0 \tag{22}$$

The (average) certainty-equivalent discount rate, r^{PV}, in this setting is:

$$\exp(-r^{\mathrm{PV}}t) = E[\exp(-\tilde{r}t)] \Rightarrow r^{\mathrm{PV}} = -\frac{1}{t}\ln[E[\exp(-\tilde{r}t)]] \tag{23}$$

which is declining over time (t) as described above.

Alternatively, imagine that we want to maximise the expected net future value (ENFV), i.e. we wish to rank our projects on the basis of maximising the value of assets that accumulate to future generations. Under an ENFV rule, a project is efficient if:

$$\mathrm{ENFV} : Z - 1E[\exp(\tilde{r}T)] \geq 0 \tag{24}$$

In this case the certainty equivalent per period interest rate, r^{FV} that produces the same outcome as the random interest rate is that which satisfies:

$$\exp(r^{\mathrm{FV}}t) = E[\exp(\tilde{r}t)] \Rightarrow r^{\mathrm{FV}} = \frac{1}{t}\ln[E[\exp(\tilde{r}t)]] \tag{25}$$

Clearly, $r^{\mathrm{PV}} \neq r^{\mathrm{FV}}$. Furthermore, r^{FV} is *increasing* over time and converges to the highest possible value of $\tilde{r}$ as $t \to \infty$. Hence, Gollier claims, when we rank projects by ENFV the socially efficient discount/interest rate is increasing over time.

So, confusingly, whereas in the absence of uncertainty the two decision criteria are equivalent, once uncertainty regarding the discount rate is introduced the appropriate discount rate for us in CBA depends upon whether we choose ENPV or ENFV as our decision criterion. In the former case, discount rates are declining and in the latter they are rising through time. It is not immediately clear which of these criteria is correct.

Gollier (2004a) explains that the two criteria differ in their temporal allocation of residual risk. Using ENPV implies that the present generation (strictly, $t = 0$) bears the risk. This is because, once the discount rate is realised (r) the NPV may or may not be positive. Since the payoff in the future (Z) is certain, any residual losses are borne by the present generation. It is as if they have a secure payoff for future generations but a random payment in the present (Gollier 2002a). For example, if the ENPV equalled zero, but the realised discount rate is greater than the certainty equivalent rate: $r > r^{\mathrm{PV}}$, the project is not viable ex post, and investors must internalise the opportunity cost. The symmetric argument to this is the case where ENPV < 0 and $r < p_{\mathrm{v}}$. However, using ENFV implies that future generations bear the risk. The present generation makes a certain contribution to the project (£ 1), but the rate at which the fund accumulates, and hence the outcome in the future ($\exp(\tilde{r}T)$), is uncertain before the realisation of $\tilde{r}$. Any shortfall is borne by the future.

Consequently, so the argument goes, choosing between these two decision criteria under uncertainty appears to be solely a question of the temporal allocation of risk. Given the risk neutral environment we cannot appeal to risk preferences in order to make this decision. Gollier argues that economic theory provides no guidance in the Weitzman set-up since current and future preferences for risk are effectively assumed away. However, the financial literature concerning the yield curve is replete with such considerations and in a number of subsequent papers Gollier returns to this literature to describe the role of risk preferences in determining DDRs (Gollier 2002a, b, 2004b). The following section describes these contributions.

Hepburn and Groom (2004) provide another perspective, taking as their starting point the observation that it is curious that the temporal allocation of risk should be so important in a risk neutral environment. They define a more general decision criteria, which they call the Expected Net Value (ENV) criteria, of which ENPV and ENFV are special cases, and show that it is the choice of the evaluation date/base year or 'temporal numeraire' rather than the allocation of risk that is important. The ENV of the project described above is defined as follows:

$$\mathrm{ENV}_\tau : ZE[\exp(-\tilde{r}(T-\tau))] - E[\exp(\tilde{r}.\tau)] \geq 0 \tag{26}$$

where τ represents the evaluation date for the valuation of costs and benefits. It is easy to see that if $\tau = 0$ then this criteria collapses to the ENPV in (22) and of $\tau = T$ it collapses to the ENFV rule in (24). The discount rate associated with this decision criterion is:

$$r^{\mathrm{ENV}}(t,\tau) = -\frac{1}{t-\tau}\ln(E\exp(-\tilde{r}(t-T)))$$

They show that, contrary to Gollier's analysis, r^{ENV} is declining in continuous time (t) but increasing in the temporal numeraire (τ). This interpretation suggests that regardless of the decision criterion, that is, for any given base year τ, where the discount rate is uncertain the socially efficient certainty equivalent discount rate (r^{ENV}) is declining over time. This suggests that Weitzman's (1998) analysis holds *no matter which generation bears the risk*. Nonetheless, Gollier (2004a) is correct to assert that the Weitzman (1998) is arbitrary since no underlying growth process or risk preferences are specified.

4.3. THE EFFECT OF UNCERTAIN GROWTH (g) ON THE SOCIAL TIME PREFERENCE RATE (δ)

In a deterministic world we noted that there are two underlying characteristics of individual preferences' which determine the social rate of time preference, δ; (i) pure impatience, ρ, and (ii) the desire to smooth growing wealth over time, θ. These are the consumption based determinants of the discount rate. In a competitive equilibrium individual preferences to discount the future are balanced against the risk-free market rate of return, r. The marginal benefits of consumption and saving are equated. Where there are frictionless financial markets, if the risk free rate of return determined in this way is used as the test discount rate for public projects the result will be an optimal level of investment (Gollier 2002a).

The difficulty in the long run is the absence of financial assets whose maturity extends to the horizon associated with the new types of projects and policies that the government is faced with, e.g. global warming. Government bonds, for example, do not extend beyond 40 years in general. In the absence of a measure of the long run discount rate determined by financial markets, Gollier (2002a, b, 2004b) turns to economic theory to provide some answers. Where proposition 1 showed the importance of these consumption based determinants of the discount rate in the deterministic world, Gollier's contributions analyse the role of these determinants when growth is uncertain. In this way he departs from the risk neutral framework of Weitzman (1998) and examines the role of risk preferences. Cast in this light, the schedule of socially efficient discount rates is determined by reference to individual preferences for risk, their evolution over time, the distribution of random growth and the analysis of the social rate of time preference, δ.

Gollier uses the framework of a 'tree economy' (Lucas 1978) in which growth is uncertain and represented by $\tilde{g}$ in order to look at the determinants of the equilibrium interest rate.[34] The growth rate of the economy is taken as the 'primal' of the model rather than the risk free rate itself, as in the case of Weitzman (1998). As in Section 2, agents make saving and consumption decisions to maximise their expected utility, $E[u(c)]$, in each period of time, t, given their expectation of future growth. Following Gollier we illustrate the arguments in discrete time. The first order condition for expected utility maximisation provides us with the determinants of the short-term risk free interest rate, $r(c)$, in this economy and can be written as:

$$1 + r(c_t) = \frac{u'(c_t)}{\beta E[u'(c_t(1 + \tilde{g}_{t+1}))]} \tag{27}$$

See Appendix C for the derivation. Equation (27) says that utility maximising individuals will equate the ratio of current and future expected marginal utility to the short-term (gross) interest rate, where future utility is discounted by the rate of pure time preference, that is $\beta = \frac{1}{\rho} - 1$. There is no productive sector in this model, therefore the risk-free rate represents the preference-based determinants of the discount rate.

The effect of certain growth upon the short term risk free rate has been described above. Gollier extends this analysis to describe the effect of uncertain growth on the short- and long-term behaviour of the discount rate. One point is immediately clear. Uncertainty in growth will reduce the discount rate when the marginal utility of consumption is convex, in which case Jenson's inequality holds: $E[u'(c(1 + g_{t+1}))] \geq u'(E[c_t(1 + g_{t+1})])$. This introduces another economic reason why individuals discount the future. Faced with uncertainty about future income levels, individuals will value additional units of consumption in the future and will save for precautionary reasons, resulting in a reduced risk free rate (Kimball 1990; Gollier 2001).

To recap, there are now three main characteristics of individual preferences that determine the risk free rate:(1) pure time preference, ρ. (2) the wealth effect reflected by θ, and (3) precaution: the desire to engage in precautionary saving in the face of uncertain income growth. The latter is reflected in the degree of convexity of marginal utility of consumption and hence is dependent upon the third derivative of utility. Individuals are said to be *prudent* when marginal utility is convex: $u'''(.) > 0$ (Kimball 1990).

In order to quantify the effects of these different determinants of the discount rate it is useful to augment the Ramsey rule. Appendix D shows that the associated expression for the risk free rate under uncertainty is:

$$r = \rho + \theta E[g_{t+1}] - 0.5\mathrm{var}[g_{t+1}]\theta P(c) \tag{28}$$

Determinants (1)–(3) are represented on the RHS of (28), respectively. The term $P(c) = \frac{u'''}{u''}y$ is a measure of *relative prudence* and is distinct from preferences for consumption smoothing and risk aversion, which is reflected once more by θ (Kimball 1990). Hence, economic theory states that the equilibrium risk free rate is decreased under uncertain growth when agents are 'prudent' (when $u''' > 0$.), and increased by the desire to smooth growing consumption over time. Consequently, the overall effect depends upon the balance between the prudence effect (the third element) and the wealth effect (the second element).

Equation (28) represents the short-term risk free rate: e.g. the return at t of a bond that yields a cash flow at time $t+1$. However, the thrust of this discussion concerns the nature of the long-run risk free rate for use in CBA. The analysis can be extended to the long-run in a fairly straightforward manner. The per-period rate of return evaluated at time t of an asset which matures at time, $t+n$, can be defined as a simple extension of equation (28):

$$(1 + r_{ct})^n = \frac{u'(c_t)}{\beta^n E\left[u'\left(c_t \prod_{t+1}^{t+n} \left(1 + \tilde{g}_{t+1}\right)\right)\right]} \tag{29}$$

where the denominator represents the value of marginal utility at time given the expected accumulation of growth between t and $t+n$. Notice that when $n = 1$, equation (29) is the same as equation (27). Equation (29) effectively characterises the yield curve: the plot of the term structure against time. This is naturally of interest since it tells us the discount rate that should be applied in CBA for costs and benefits that occur at each date.

In general, the interest rate will depend upon the maturity. For example, it is well known that if agents in period t expect growth in period to be significantly lower (higher) than growth in period $t+1$, then the yield curve will be downward (upward) sloping. This can be deduced from equation (29). This outcome is analogous to the discussion concerning deterministic growth in Section 3. However, in order to control these effects, Gollier (2002a) undertakes his analysis in a context in which growth is expected to be similar across periods. The shape of the yield curve then depends upon the nature of the preferences held by individuals and the subsequent temporal balance between wealth effects and prudence effects.

Gollier (2001, 2002a, b) presents several results of interest. Firstly, when individuals display Constant Relative Risk Aversion (CRRA) the yield curve is flat and the prudence and wealth effects exactly compensate one another. This corresponds to the conventional situation in which the discount rate for CBA remains constant for all time. Secondly, when it is assumed that there is no possibility of recession in the future, and individuals display Decreasing Relative Risk Aversion (DRRA), the yield curve is downward sloping. Then the risk free rate is declining over time and thus, the discount rate for CBA declines over the time horizon of the project. Lastly, when the prospect of recession is introduced the conditions for a declining yield curve become highly specialised. For example, if there is only a risk of recession in the long run, the yield curve is declining only if individuals display both DRRA *and* Increasing Absolute Prudence (IAP). This means that $P'(c) > 0$ (there are a number of additional necessary conditions for this to hold – for details see Gollier (2002b)). This represents a distinct class of utility functions with restrictions upon the 4th derivatives. Furthermore, if the risk of recession is extended to all future periods, short-run and long-run, a declining yield curve requires restrictions on the 5th derivatives of the utility function. As Gollier himself states, there is little hope that such conditions can be tested in the near future.

The complexity of the analysis is dependent upon the assumptions concerning the probability distribution of growth and the inter-temporal relationships. For the purpose of the analysis above, Gollier (2002a, b) assumes that the growth shocks are independently and identically distributed. Although this is unrealistic, it avoids the complications associated with the analysis of serially correlated shocks. In more recent work, Gollier (2004b) provides an analysis of the long-term discount rate in which these assumptions concerning serial correlation are relaxed. He finds that where there is positive correlation between the expected value of future growth and the short term growth rate, a downward sloping yield curve requires only that the representative agent is prudent, that is $u'''(.) > 0$. Clearly these conditions on preferences are less restrictive than in the *i.i.d* case assumed above. A number of other results are presented for different assumptions concerning the serial correlation of growth rates. One interesting example allows for a stochastic process which switches randomly between high and low growth regimes with Poisson events. Another reflects the approach of Weitzman (2004) and includes Bayesian learning as the source of positive serial correlation. In both cases, DDRs emerge if the representative agent has CRRA preferences. Furthermore, the declining schedule is more rapid with positive serial correlation of growth rates than without.

Gollier's analysis provides some potentially testable propositions, which draw directly from expected utility theory. The formal economic foundation for the determination of long-term discount rates avoids the *ad hoc* adjustments of the discount rate common in the literature. Furthermore, the explicit treatment of risk is potentially more general that the risk neutral environment of Weitzman (1998). This approach is indeed technical and complicated, and the preferences that lead to DDRs are frequently difficult to test, but as Gollier (2004a) notes:

> *'this is probably the cost to be paid to make policy recommendations that make economic sense'* (Gollier 2004a, p. 5)

Not only are preferences of great importance here, the recent contributions to this area emphasise the importance of the assumptions concerning the distribution of random growth within and between periods. Moreover, we should remain open to the prospect that preferences and stochastic processes in society are such that the socially efficient discount rate could be decreasing, constant or increasing over time.

5. Intergenerational Equity and Sustainability

The foregoing has concerned itself with the analysis of the efficient discount rate and its behaviour over time without any real discussion about the implications for inter-generational equity and sustainability. This section reviews the research taking sustainable growth and inter-generational equity as a departure point. The main focus of the discussion is on the important contributions of Chichilnisky (1996, 1997) and Li and Löfgren (2000), both of whom explicitly introduce the notions of intergenerational equity and sustainability. Each paper models optimal sustainable economic growth and each is concerned with deriving the welfare effects of growth paths which are sustainable in the sense that they satisfy particular axioms with regard to intergenerational equity. The axioms employed imply social preferences which are 'sustainable' or 'intertemporally equitable'. Welfare is measured in terms of the utility of a social planner and, with utility as their numeraire, the discussion of discount rates concerns the utility discount rate, ρ, rather than the social rate of time preference, δ, or the social rate of return, r. Both contributions show that a declining utility discount rate is consistent with a rule whereby current (future) generations must always take into account the well-being of future (current) generations. That is, there must be no 'dictatorship' of one generation over another. In this way what Chichilnisky (1997) refers to as the 'tyranny of the present over the future' associated with constant rate discounting is overcome.

Chichilnisky (1997) introduces two axioms for sustainable development.[35] She also characterises the preferences that satisfy these axioms. The axioms require that the ranking of alternative consumption paths is sensitive not only to what happens in the present and immediate future, but also to what happens in the very long run. Sensitivity to the present means that there is no date before which events are given zero weight. Sensitivity to the long-run future means that there is no date where changes after that date do not matter, in the sense of affecting the ranking. Chichilnisky's criterion can be represented in the following objective function:

$$\max_{c,s} \pi \int_0^\infty u(c(t), s(t)) \exp(-\rho t) \mathrm{d}t + (1 - \pi) \lim_{t \to \infty} u(c(t), s(t)) \tag{30}$$

Instantaneous utility u (.) is a function of consumption (c) and the resource stock (s) at each time period (t), while $\exp(-\rho t)$ is the conventional exponential utility discount factor. u (.) is assumed to be the same for all dates so that generations are assumed to be the same in the way they rank alternatives.

The limit term can be interpreted as the well-being of generations in the far distant future. Chichilnisky's criterion thereby balances the discounted utilitarian approach, with an approach that ranks paths of consumption and natural resource use according to their long-run characteristics, or sustainable utility levels. Notice that $\pi \in [0, 1]$ can be interpreted as the weight that the decision maker applies to each component of the criterion, with π providing the weight given to the present generation, and $(1 - \pi)$ representing the weight placed upon the future generation.

Dasgupta (2001) has criticised this approach, noting that there is a way in which all generations can have their cake and eat it too.[36] Suppose the current generation devises a plan that maximises only the integral part of the maximand in equation (30). It simultaneously announces its intention to abandon that plan at some date in the distant future, at which point it will switch to a plan that then maximizes only the asymptotic part of the maximand. The farther this switching date is in the future, the more nearly the integral part will be maximized. But there will always be an infinite number of dates after the currently planned switching date, and hence it will always be possible to increase welfare by postponing the switching date.

In contrast to Chichilnisky (1997), Li and Löfgren (2000) assume society consists of two individuals, a utilitarian and a conservationist. The utility functions of these two individuals are identical, although they employ different utility discount rates. The objective function employed by Li and Löfgren is:

$$\max U = \pi U_1 + (1 - \pi) U_2 = \int_0^\infty u(c(t), s(t)) D(t) \mathrm{d}t \tag{31}$$

where

$$U_1 = \int_0^\infty u(c(t), s(t)) \exp(-\rho_U t) \mathrm{d}t \tag{32}$$

$$U_2 = \lim_{\rho_C \to 0} \int_0^{\infty} u(c(t), s(t)) \exp(-\rho_C t) \mathrm{d}t \tag{33}$$

where $D(t)$ is the discount factor. The utilitarian, who wants to maximise the present value of his utility (U_1), has a rate of time preference equal to ρ_{U}. The conservationist, with utility U_2, has a rate of time preference equal to ρ_{C} and maximises her utility. The overall societal objective is to maximise a weighted sum of wellbeing for both members of the society, given their different respective weights upon future generations. The effective utility discount rate in Li and Löfgren is given by:[37]

$$\rho(t) = -\frac{1}{t} \ln\{(1 - \pi) \exp(-\rho_{\mathrm{C}} t) + \pi \exp(-\rho_{\mathrm{U}} t)\} \tag{34}$$

If the conservationist discounts the future at a rate of zero, $\rho_{\mathrm{C}} = 0$, the corresponding discount factor is:

$$D(t) = (1 - \pi) + \pi \exp(-\rho_{\mathrm{U}} t) \tag{35}$$

In the distant future when t is large, (35) has a minimum value of $(1 - \pi)$, the weight attached to the conservationist, or future generations. It is in this way that the effective discount rate can be thought of as declining over time to zero. Thus, unlike the utilitarian discount function, which tends to zero as time reaches towards infinity, the weighted discount function tends to the weight for the far distant future. Hence, Li and Löfgren's model results in a positive welfare weight for the conservationist and there is no dictatorship of present over future generations. As the utilitarian's welfare level is explicitly considered, there will also not be any dictatorship of the future over the present. Thus, the model explicitly considers intergenerational equity. Within this framework, the conservationist will dominate the far-distant future. Therefore, the discount rate will be a declining function of the time horizon.

6. Hyperbolic Discounting

We have seen some of the normative and theoretical arguments for DDRs in the discussion above. In this section we concern ourselves with the considerable empirical and experimental evidence of how individuals discount time.

6.1. EVIDENCE

Over the last couple of decades, increasing evidence from experiments conducted by economists and psychologists in the lab and the field suggests that people use a declining discount rate in making intertemporal choices. Researchers typically ask subjects to choose between a set of delayed rewards, and construct the shape of the discount function from their responses. Harris and Laibson (2001) note that a large number of such experiments has been conducted, with a variety of rewards such as money, durable goods, sweets, relief from noise and so on.[38] The results from these experiments suggest quite strongly that the discount rate applying to consumption trade-offs in the present is higher than that applying to trade-offs in the future. In other words, individuals are more sensitive to a given time delay if it occurs closer to the present than if it occurs farther in the future.

There are some dissenting voices, however, Read (2001) and Rubinstein (2003) offer other interpretations of the empirical evidence, Rubinstein (2003) presents his own experimental evidence that is not consistent with either constant or hyperbolic discounting, but is consistent with a decision-making procedure based on *similarity relations*.[39] This procedure assumes that individuals ignore small differences and focus on large differences when comparing two alternatives. Read (2001) argues that the so-called evidence of hyperbolic discounting is in fact evidence of *sub-additive discounting*, where discounting over a given period is greater when the period is divided into subintervals than when it is left undivided. This implies an inverse relationship between the discount rate and the size of the delay. In other words, Read (2001) argues that the discount rate is not a function of relative location in time, as proponents of hyperbolic discounting suggest, but is rather a function of the size of the time delay. Finally, Mulligan (1996) argues against hyperbolic discounting on the basis that hyperbolic discounters leave themselves open to exploitation on the markets by 'Dutch books'. People with that tendency, he argues, would rapidly learn to correct their ways. While this logic might hold on futures markets, we would doubt that hyperbolic discounting at an individual day-to-day level would be damaging enough for people to modify their behaviour. So although the jury is still out on the precise explanation for the empirical evidence, the support for hyperbolic discounting is relatively strong.

Loewenstein and Prelec (1992) proposed that hyperbolic preferences could be modelled by a generalised hyperbolic discount function of the form:

$$D(t,\tau) = (1 + \varpi(t-\tau))^{-\xi/\varpi} \quad \text{for } \varpi, \xi > 0 \tag{36}$$

where the coefficient ϖ determines the extent of departure from exponential discounting. As $\varpi \to 0$, we obtain standard exponential discounting. When ϖ is large, $D(t)$ approximates a step function. Note that in the literature, 'hyperbolic discounting' has increasingly been employed to refer to *any* declining discount rate, not just discount functions that follow a hyperbola.

Variations on the hyperbolic theme have discount rates that are non-zero in the long run.[40] In discrete time, the hyperbolic function can be approximated by a quasi-hyperbolic function, used originally by Phelps and Pollak (1968), later by Akerlof (1991) and popularised by Laibson (1997). It can be represented as a series of discount factors $\{1, \beta\zeta, \beta\zeta^2, \beta\zeta^3, \ldots\}$,[41] where the implicit long-run discount rate is non-zero.

6.2. IMPLICATIONS OF HYPERBOLIC DISCOUNTING

Because hyperbolic and quasi-hyperbolic discounting imply a time-varying discount rate, they can result in time-inconsistent preferences.[42] Time inconsistency implies that plans made today will not be carried out tomorrow unless a mechanism to commit the later self can be implemented.

Because of this feature, Akerlof (1991) suggested that hyperbolic discounting might have useful applications to model procrastination, drug addiction, under-saving, and organisational failure, inter alia. In the last five years, more detailed hyperbolic models have emerged and have been applied to an enormously large range of economic phenomena. Laibson (1994, 1997, and Laibson et al. (1998) have considered the problem of under-saving in depth. Harris and Laibson (2001, 2003) extend this work to model buffer-stock saving. Retirement timing is considered by Diamond and Koszegi (1998). Drug addiction is examined by Gruber and Koszegi (2001), while O'Donoghue and Rabin (1999a,b) and Benabou and Tirole (2000) have examined procrastination. Barro (1999) shows that, under certain circumstances, optimal growth trajectories under hyperbolic discounting are observationally equivalent to those under exponential discounting. In the environmental sphere, Cropper and Laibson (1999) consider the effect of hyperbolic discounting in project evaluation and qualitatively consider the arguments for applying a lower discount rate to environmental projects.

7. Practical Implications for CBA

7.1. A BRIEF SUMMARY SO FAR

The preceding sections have provided several rationales for DDRs. In a deterministic world, DDRs can arise as a result of known changes in the growth rate, changes in consumption smoothing/risk aversion, increasing expenditures on the environment in the presence of environmental externalities, or increases in marginal WTP for the environment. Clearly each rationale has its strengths and weaknesses. Additional motivations emerge once uncertainty is considered. Uncertainty of the discount rate itself provides a simple and intuitive approach in a risk neutral environment. In the presence of uncertain growth Gollier shows that DDRs depend upon preferences for risk and prudence, and higher order moments of the utility function. Regardless of whether it is the discount rate or the growth rate that is uncertain, DDRs depend upon the nature of the underlying probability distribution. DDRs also emerge from the specification of a 'sustainable' welfare function à la Chichilnisky (1997) and Li and Löfgren (2000). Lastly, there is considerable empirical and experimental evidence to show that individuals are frequently hyperbolic discounters.

In sum, the practitioner is left with a confusing array of rationales for DDRs and little guidance as to the implications of employing them nor how to construct a workable schedule. In the following Sections we address these points directly.

7.2. PARAMETER IDENTIFICATION

Once a rationale has been subscribed to, implementation requires the practitioner to identify a particular set of parameters, i.e. an answer to the second question raised: what trajectory should a DDR follow? The required parameters for determining the time invariant discount rate in the deterministic case have been discussed extensively elsewhere (see, for example, Pearce and Ulph 1999) and are well understood. Here, we focus upon the application of the more recent contributions.

Horowitz (2002) reduces the discussion of the discount rate to a valuation problem: valuing future preferences. This requires analysis of the effects on WTP of changes in income and environmental quality. However, we noted that there are strong arguments for keeping 'the' discount rate separate from valuation of goods and services. Weitzman's deterministic model (Weitzman 1994) requires information on the trend of the proportion of income spent on environmental goods (environmental protection), and the effectiveness of this expenditure in maintaining environmental standards in order to derive a DDR. These can be thought of as aggregate statistics in his model, and the theory is perhaps easily applied in this sense. However, the mechanism by which discount rates are affected, although intuitive, is quite particular.

In order to implement the approach suggested by Weitzman (1998), it is necessary to characterise the uncertainty of the interest rate. In general terms this amounts to defining a probability distribution for the future discount rate, and its behaviour over time. In this sense there are two ways in which we can interpret the example in Table 1. Firstly, it could represent the thought experiment of Weitzman (1998), in which we are currently uncertain about interest rates, and yet the interest rates will persist indefinitely ex post realisation. In this sense we have a probability distribution for the current uncertainty, which assumes that interest rates of 2% and 5% are equally likely, and we employ this distribution for all future periods. Uncertainty is therefore regarded as existing from day one, and all that is required is the current probability distribution of the discount rate.

In a further article, Weitzman (2001) takes precisely this approach. In order to establish the probability distribution for the socially optimal discount rate he undertakes a survey of over 2000 academic economists, and a so-called 'blue ribbon' selection of 50, as to their opinion on the constant rate of discount to use for CBA. The responses were distributed with a gamma distribution with mean 4%, and standard deviation 3%, providing an *ad hoc* working assumption to determine the schedule of DDRs. The assumption implicit in the use of the gamma distribution is that there is uncertainty in the present about the interest rate in the future and that when uncertainty is resolved the realised interest rate will persist forever.

Newell and Pizer (2003) take an alternative view. Rather than assuming that uncertainty in the discount rate represents a current lack of consensus about *the* discount rate, they consider the interest rate as a stochastic process, that is, there is uncertainty in the future about interest rates. N&P characterise this uncertainty using time series econometric modelling of the autocorrelation process of interest rates. The estimated model is used to forecast future rates based upon their behaviour in the past. From these forecasts they derive numerical solutions for the CE. In doing so they are also able to provide a test of another assumption important to the Weitzman (1998) result, namely the presence of persistence of discount rates over time. They compare the discount rates modelled as a mean reversion process to a random walk model, and find support for the latter. As we shall see in Section 8, the greater persistence of interest rates following a random walk compared to a mean reverting process has important implications for the value of long-term costs and benefits since the decline in discount rates is more pronounced. However, using UK interest rate data Groom et al. (2004) provide a more thorough econometric analysis of the extent to which uncertainty in the future causes DDRs and find that model specification is crucial to the analysis, not least because of the distributional assumptions contained therein. Indeed, they find little evidence of the persistence noted by Newell and Pizer, suggesting that in the UK context the effect of future uncertainty upon the valuation of global warming damages is minimal. We return to this issue in Section 8.

In addition to determining the probability distribution, it is necessary to make some assumption concerning the point in time at which uncertainty concerning the discount rate begins. Weitzman (1998) suggests employing the declining discount rate at some period T, beyond which uncertainty is said to begin, but gives no particular guidance as to how to identify T. It seems reasonable to suggest that the limits of financial markets define a useful starting point for uncertainty. Government bonds generally have the longest maturity, dates and reflect the market evaluation of future discount rates up to around 30 years in general. Hence, $T = 30$ could be the point beyond which the certainty equivalent analysis should begin (e.g. as argued in OXERA 2002). Newell and Pizer (2003) implicitly assume that the uncertainty begins immediately, although, with high levels of persistence, the forecast remains relatively constant over the short-term.

The rationale for declining discount rates provided by Gollier (2002a, b) is perhaps the most theoretically rigorous of all the contributions, given the indeterminacy surrounding Weitzman (1998). But determination of the trajectory requires very specific information concerning the preferences of current generations at the very least, and, in the long-run, the preferences of future generations.[43] These parameters include the aversion to consumption fluctuations over time, the pure time preference rate, and the degree of relative risk aversion. For the case with zero recession, restrictions on the 4th and 5th derivatives of the utility function become necessary. In addition, the probability distribution of growth needs to be characterised in some way. Clearly, the informational requirements of the Gollier approach could be daunting.

Implementation of the Li and Löfgren and Chichilnisky approaches requires the identification of several other parameters, including specification of the utility discount rate for the 'utilitarian', and perhaps more importantly, the relative weight to be assigned between 'conservationist' and 'utilitarian' preferences. Although the selection of this weighting might appear to be relatively arbitrary, it makes the trade-off between present and future generations explicit, and could possibly be determined by an appropriate political process.

7.3. TIME INCONSISTENCY

We remarked in Section 6 above that declining utility discount rates may produce time inconsistent planning. Indeed, hyperbolic discounting has been so successful in the behavioural economics literature precisely because time inconsistent behaviour helps to explain phenomena such as procrastination and addiction. Generally, well-being is not maximised in such situations.

Faced with this potential for dynamic inconsistency, a government without a commitment mechanism can formulate policy in a 'naïve' or 'sophisticated' manner. Neither situation is satisfactory. The sophisticated government takes into account the fact that future governments will have an incentive to deviate from its optimal (committed) policy. The situation may be modelled as an intertemporal game played with its successors as per

Phelps and Pollack (1968). In the Nash equilibrium, the government makes policy as the best response to successive government's best responses. It, therefore, manages to retain credibility and, as Barro (1999) and Karp (2003) illustrate, time-consistency. However, the Nash equilibrium is not Pareto optimal. Interestingly, under certain conditions discussed in Barro (1999) this Nash equilibrium policy ends up being equivalent to a policy that would have been constructed using a conventional exponentially declining discount rate. In contrast, the 'naïve' government presses ahead regardless with dynamically inconsistent policy, ignoring the fact that future governments will find its policies to be sub-optimal. This is also clearly sub optimal, as from the perspective of the current 'naïve' government, its optimal policy will not be adhered to.

Some writers do not see this to be a problem. For instance, Henderson and Bateman (1995) argue that the process of changing the discount rate as time moves on as legitimate. They assert that people see themselves living in relative, rather than absolute, time. Revising and re-evaluating plans as time moves on is not only consistent with behavioural studies, but with the value judgement that what ought to be done by way of discounting should reflect what people actually do. However, for others, ourselves included, it is not clear that empirical evidence of individual preferences is entirely relevant to the social discount rate. A Humean would contend that simply because people *do* discount the future hyperbolically does not mean that they *should*, nor does it imply that this is advisable practice for government. On the other hand, one might argue that if people's preferences count, and if people employ hyperbolic discounting, those preferences must be integrated into social policy formulation. The utilitarian leaps effortlessly from 'is' to 'ought' statements because of the assumption that behaviour reflects preferences.

Nevertheless, this assumption has been questioned not only by philosophers but also by economists such as Feldstein (1964). Indeed, more recently, a literature on 'optimal paternalism' is developing which suggests, amongst other things, that governments may be justified in intervening not only to correct externalities, but also to correct 'internalitiés'; behaviour that is damaging to the actor. Recent work on sin taxes by O'Donoghue and Rabin (2003) provides an example of this type of approach. Whether or not one supports a paternalistic role for government, however, the wisdom of adopting a discount function that explains procrastination and addiction for social policy is questionable. Our overall conclusion is that although the evidence that individuals employ hyperbolic discounting is strong, the argument that governments should do likewise is weak.

Heal (1998) takes a different tack in arguing that time consistency is not significant. He notes that at an individual level, individuals at different stages of life might appropriately be thought of as different people, so that requiring time consistency is somewhat stringent. We know from the theory of preference aggregation that societies generally satisfy weaker rationality conditions than their composite individuals, so from a social choice perspective time consistency is a ‘most unnatural requirement’. While this is correct, the consequences of time inconsistency at a social level, just as the individual level, can be particularly severe. Hepburn (2003), for instance, shows that a naïve government employing a hyperbolic (declining) discount rate in the management of a renewable resource can unwittingly manage the resource into extinction.

Newell and Pizer (2003) argue that they are able to ‘circumvent’ the time inconsistency problem. In their model, the decline in future discount rates follows from uncertainty about future events rather than an underlying preference for a deterministically declining discount rate. But it is not clear that this circumvents the problem at all. Irrespective of the theoretical or empirical basis for the use of declining discount rates, if they are used naïvely a time inconsistent policy will result. As Hepburn (2003) notes, building awareness of the problem, thereby encouraging the use of declining rates in a sophisticated or committed manner, is surely better than assuming it away.

There is no easy resolution of the time-inconsistency problem. Incongruence, or dynamic inconsistency, results in consumption and savings plans that are sub-optimal for all generations. Heal (1998) proves that almost all types of declining discount rates are time inconsistent, so the extent of the problem is certainly significant. As a practical matter, however, the dynamic inconsistency inherent in declining discount rates may not be any more troubling than policy inconsistencies and changes that are prompted by external shocks or political shifts. More work is needed in this area.

8. Implications of Declining Discount Rates: Some UK Case Studies

In this section, we investigate the implications of DDRs for policy. We employ some of the methodologies described above to two issues: climate change and nuclear power. This involves an application of the Weitzman/Newell and Pizer (2003) approach to UK interest rate data.

8.1. UNCERTAINTY OF UK INTEREST RATES IN THE FUTURE

In this section, we describe a declining discount rate schedule derived from the application of the estimation procedure used by Newell and Pizer (2003) (N&P) to UK interest rate data. In short, interest rates are forecasted over a period of 400 years using the results of an estimated reduced form random walk model. The schedule of certainty equivalent discount rates is derived from the simulation of up to 100,000 interest rate forecasts and use of Weitzman's definition of the certainty equivalent discount rate (CER). We also present the results of a 'state-space' model applied to the UK data, which takes into account the possibility of structural breaks and allows for the autocorrelation process driving interest rates to change over time. These are important determinants of discount rate uncertainty, which represent a more appropriate methodology for forecasting discount rates for the very long-term and a departure from N&P. The details of the econometric models used are shown in Appendix D.

Figure 1 compares the schedule of the certainty equivalent discount factors derived from the two forecasted models to the discount factor that is derived from discounting at a flat rate of 3.5%. It is easy to see that schedule of certainty equivalent discount factors derived from the state space model is higher than those derived from the N&P method, whilst the latter is fractionally higher than with constant discounting. These results are similar to those of N&P for the US: interest rate uncertainty in the UK provides a rationale for DDRs to be employed in project appraisal. However, there are two further practical points that arise from this analysis. Firstly, in applying N&P, we fail to establish the existence of persistence, indicating that the mean reverting model is more appropriate than the random walk model. This is the inverse of N&P's finding for the US. Secondly, model selection is important. The state space model is introduced to improve upon the misspecified mean reverting model (See Appendix D for details). The model and results show the importance of introducing flexibility into the characterisation of uncertainty e.g. accounting for structural breaks and autocorrelated coefficients. Indeed there are a number of other empirical issues that need to be addressed before an acceptable sche-

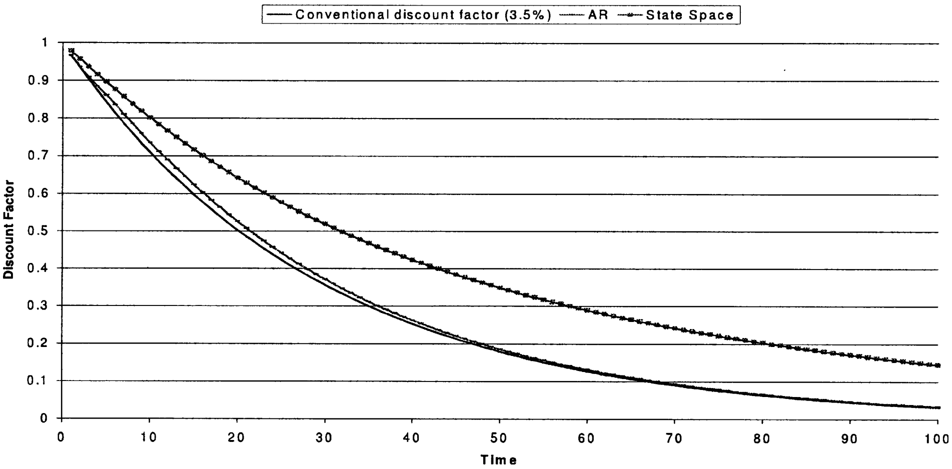

Figure 1. Conventional and empirical discount functions.

dule can be determined empirically. These issues are discussed at length in Groom et al. (2004). The implications of these estimates are described below.

8.2. SOCIAL COST OF CARBON

The social cost of carbon is an estimate of the present monetary value of damage done by anthropogenic carbon-dioxide emissions. The UK has an ‘official’ value of this shadow price (Clarkson and Deyes 2002) at pounds 70 per tC, although the validity of the number is disputed (Pearce 2003) and the official value is under review at the time of writing. Self-evidently, higher values of the social cost of carbon imply that investment in climate change mitigation is more attractive. The discounting framework employed has a significant impact upon such estimates. It is obvious, for instance, that a lower (constant) discount rate will increase the present value of the marginal damage from emissions. For example, the marginal damage values from the Fund 1.6 model (Tol 1999) increase from $20/tC to $42/tC to $109/tC, as the discount rate declines from rates of 5% to 3% to 1%, respectively.

In order to illustrate the difference between the various discounting frameworks on the social cost of carbon, we start with an approximate profile of the economic damage done by one tonne of carbon emissions in 2000, shown in Figure 2. This is the profile of damages generated by the DICE model of Nordhaus and Boyer (2000). Applying the various discounting regimes to this damage profile over the next 400 years results in estimates of the social cost of carbon presented in Figure 3. For the 200-year period, the estimates vary from approximately pounds 2.50/tC at a 6% flat discount rate, to about pounds 20.50/tC under a discounting regime based on the Li and Löfgren approach.

Increasing the time horizon from 200 to 400 years makes no difference when constant discount rates are employed, because the discount factor approaches zero well before the 200 year mark. In contrast, marginal damage estimates under declining discount rate regimes are noticeably larger when the time horizon is extended to 400 years.

Furthermore, the application of N&P’s methodology to UK data increases the 400 year estimates of marginal damage costs by a mere 4.3% compared to the constant discounting regime. This contrasts with N&P’s finding of an 84% increase. This reflects the lower level of persistence found in the UK case compared to the US, resulting in the mean reverting model being more appropriate than the random walk model of N&P. The state-space model leads to a 150% increase in the value of marginal damage. This model is well specified and is therefore more credible. The magnitude of

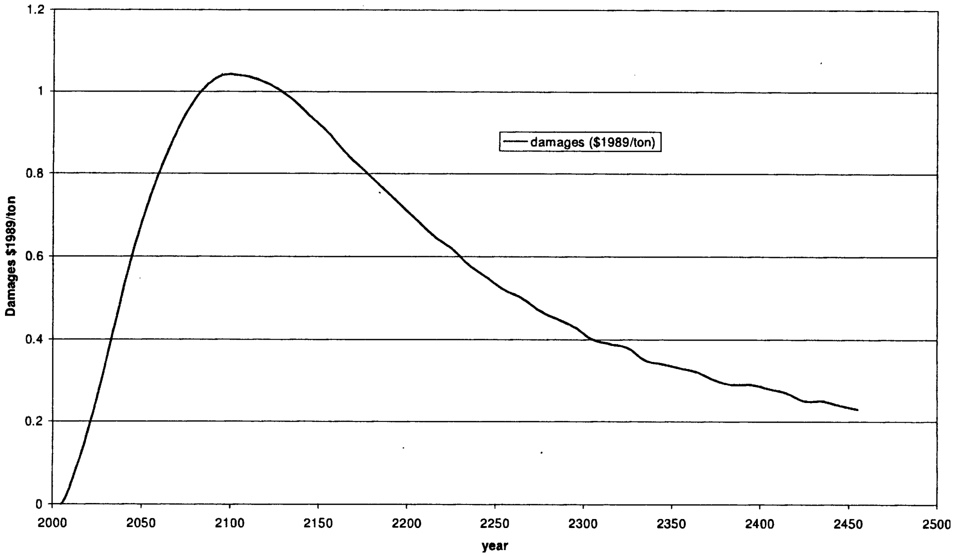

Figure 2. Profile of carbon damages from the DICE model.

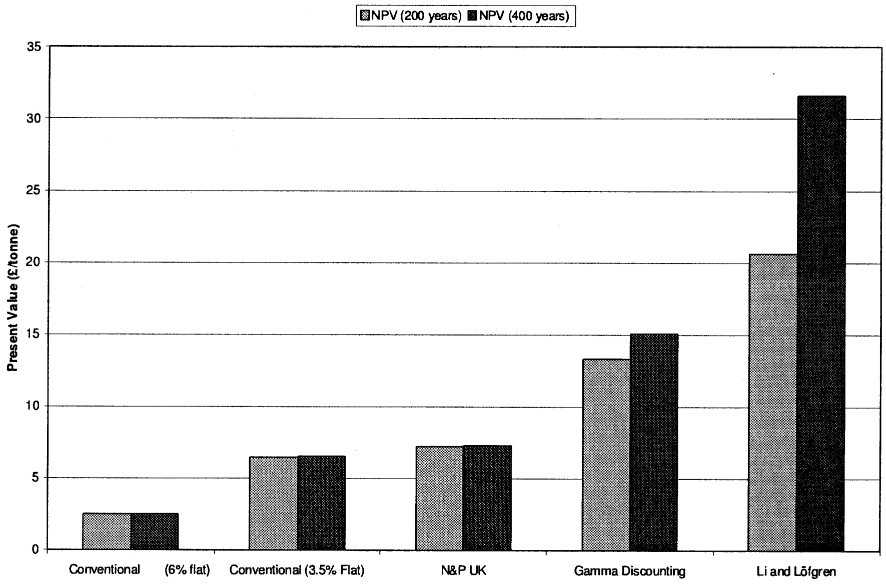

Figure 3. The discounted value of carbon mitigation.

the differences reflects once more the practical implications of model selection in determining the schedule of CER.

This illustration suggests that estimates of the social cost of carbon are likely to at least double if declining discount rates are employed. This would have formidable implications for policy in several areas. For example, a higher social cost of carbon would make it more likely that commitments to Kyoto targets would pass a cost-benefit test (Pearce 2003).

8.3. NUCLEAR POWER

New nuclear build in the UK is still being considered as an option to ensure security of energy supply and meeting long run climate change targets. In 1998, the House of Commons Trade and Industry Committee recommended that: ‘A formal presumption be made now, for purposes of long-term planning, that new nuclear plant may be required in the course of the next two decades.’ This recommendation has been supported by a joint working group of the Royal Society, and the Royal Academy of Engineering. More recently, the Performance and Innovation Unit (Performance and Innovation Unit, 2002a) recommended that the nuclear option should be kept open.

These recommendations are based upon conventional assessments of the economics of new nuclear build, which are ‘relatively insensitive to back end costs.’ (Performance and Innovation Unit 2002b). In other words, the present-value of decommissioning costs is insignificant using conventional discounting. However, the present-value costs of decommissioning approximately double if declining discount rates are employed. From PIU (2002b) we assume a construction cost of £ 2,250/kW in 2000, and a load factor of 0.85. Employing submissions from the NUCG to the 1995 White Paper on The Prospects for Nuclear Power in the UK, we assume variable operating and maintenance cost of 0.6 p/kWh, and fuel cost of 0.4 p/kWh, in 1993 money. We also assume fixed operating costs of 1.5% of construction cost. Construction occurs over 6 years, the reactor lifetime of 40 years, and decommissioning and waste management occurs over the following 70 years. PIU (2002b) state that ‘it is impossible to estimate waste management costs in any useful way at present’ due to immense uncertainty. For illustration purposes, we assume combined decommissioning and waste costs of £40/kW per year over the 70 year period, implying total decommissioning costs of £2800/kW (undiscounted).

As Table II illustrates, our calculations suggest that decommissioning costs would increase from approximately £90/kW, with a flat 6% discount rate, to £1190/kW applying the approach of Li and Löfgren. At this level, decommissioning and waste costs are a major determinant of the economic viability of nuclear power and can no longer be relegated to the realm of politics.

But there are two further countervailing effects. Firstly, a declining discount rate increases the present-value of the generation revenue earned over the 40-year lifetime of the reactor. In other words, declining discount rates reduce the weight on the initial front-end costs and increase the relative weighting on revenue earned in the future. Secondly, if an emissions tax based upon the social cost of carbon were imposed upon conventional generators, declining discount rates would improve the relative economics of nuclear generation by raising the social cost of carbon. The size of these effects, based upon the assumptions employed above, is presented in Table II.

9. Conclusions

The realisation that actions taken today can have long term consequences presents a challenge to decision makers in assessing the desirability of policies and projects. The use of the classical net present value (NPV) rule to assess the economic efficiency of policies with costs and benefits that accrue in the long term is felt by many non-economists to be particularly problematic. The welfare of future generations barely influences the outcome of such a rule when constant discount rates are used for all time. The deleterious effects of exponential discounting ensure that projects that benefit generations in the far distant future at the cost of those in the present are less likely to be seen as efficient, even if the benefits are substantial in future value terms. In this respect it appears that the present wields a dictatorship over the future. The idea of using declining social discount rates (DDRs) has emerged largely in response to these awkward implications and recently DDRs have even been accepted at an official level in the UK (HM Treasury 2003).

The approaches reviewed here are predominantly theoretical contributions to an inherently practical issue. Ultimately, the practitioner is faced with a potentially confusing array of rationales and a sense that almost any discount rate can be applied. Moreover, it is important that the practitioner is

Table II. Effects of DDRs on present values of nuclear power costs and revenues

Revenues and Costs	6% flat rate	3.5% flat rate	N&P UK	Gamma	State Space	Li and Löfgren
Revenues	2527	4062	4210	4343	5365	3853
Carbon Credit	90	228	255	528	571	1110
Capex	2054	2173	2181	2152	2238	2116
Opex	1453	2336	2421	2497	3085	2216
Decomm	90	427	497	939	1185	1192
Net present value	−980	−646	−634	−717	−572	−560

aware that the implications of employing declining discount rates are of considerable moment. Firstly, as our case studies show, there is the potential to reverse the recommendations of social cost benefit analysis in the long-term policy arena. This is especially important given the nature of this policy arena and the considerable changes that might be required in order to prevent the impact of global warming. Secondly, declining discount rates introduce time-inconsistency to the decision making process. This may turn out to be problematic for the practitioner. More importantly, the stakes are potentially very high in this arena and, to the extent that economic analysis is used on both sides of the argument in international policy-making, the analysis must be robust and well conceived.

The case for declining social discount rates is still not proven beyond doubt, despite the extremely persuasive contributions reviewed in this paper. Indeed, the use of DDRs may put us in danger of placing more weight upon potentially richer individuals in the far distant future than we place on potentially poorer present individuals. What is more widely agreed is the limited extent to which discount rates can be manipulated to simultaneously reflect the numerous underlying issues that have motivated their investigation, namely inter-generational equity, sustainability and efficiency. However, admitting a time-varying discount rate at least provides another degree of freedom."

Acknowledgements

Ben Groom, Cameron Hepburn and Phoebe Koundouri dedicate this paper to our teacher and colleague David Pearce, who passed away unexpectedly in September 2005.

We should like to thank to Jo Swierzbinski for this numerous insights and two anonymous referees for useful comments. The usual disclaimer applies.

Notes

1. When considering the marginal project or investment.
2. The shadow price or accounting price interpretation strictly refers to a decentralised economy, rather than to a social planner.
3. There are instances in which the outcome of the Ramsey planning problem is also descriptive of the equilibrium of a stylised economy. However, there are many instances where this equivalence breaks down leaving purely a normative interpretation. For further discussion on these and related issues see Phelps and Pollack (1968) and Harsanyi (1955).
4. The use of a representative agent model abstracts from the fact that individuals have different rates of pure time preference. Gollier and Zeckhauser (2003) look at collective investment decisions where pure time preference is heterogeneous among the agents within an economy.

5. $u'(.)$ represents the first derivative with respect to $c, u''(.)$ the second, etc. This notation holds throughout the paper and for other functions where no confusion arises.

   In the Ramsey model, the felicity function is also assumed to satisfy the Inada conditions: $u'(c) \rightarrow 0$ as $c \rightarrow \infty$ and $u'(c) \rightarrow \infty$ as $c \rightarrow 0$.

6. For simplicity we abstract from depreciation, population growth and technological changes here. Note also that the production function $f(.)$ is generally assumed to satisfy the Inada conditions.
7. The concepts of private and social rates are more intuitive when considering competitive, decentralised representation of this model. In that descriptive case it is also true that $i=r$ in the absence of externalities and distortions.
8. Manne (1995) and Stephan et al. (1997) compare this approach to an Overlapping Generations model (OLG) in the context of climate change policy and find that the OLG model offers little in the way of additional policy insights.
9. Both were considering a general planning problem and hence what an 'ethical' planner should think, rather than taking a descriptive view of the economy.
10. There are a number of arguments either way concerning utility discounting. Economists' arguments for $\rho > 0$ are frequently concerned with the high level of savings and the immiserisation of current generations that may result in the traditional infinite horizon model. Others suggest that since impatience is observed among individuals it should be reflected in the decision-making process. Philosophers and economists alike are not agreed that these arguments are entirely satisfactory.
11. The importance of the curvature of the utility function is also touched upon by Olson and Bailey (1981).
12. This is so if the felicity function is concave: $u'(.) > 0$ and $u''(.) < 0 \Rightarrow \theta > 0$.
13. To see this consider the following example. If corporation taxes are 50% and income taxes are 25% then if $\delta = 6\%$ then when firms invest they must pay dividends to shareholders such that they obtain a 6% return. This means that the shareholders must earn a pretax profit of 8% $(8\% * (1 - 25\%) = 6\%)$ while investors/firms must earn 16% $(16\%(1 - 50\%) = 8\%)$. That is $i = 16\%, \delta = 6\%$, and the rates are divorced.
14. The shadow price of capital is simply the present value of the future stream of consumption benefits associated with £1 of private investment discounted at the SRTP. In the case of a 2 period project yielding benefits $B_t = [B_1, B_2]$ and a private investment yielding the rate of return on private capital, r, of 16% 1 year hence, then the consumption lost as a result of the public project as a result of the £1 displaced from the private project is £1.16. This is the shadow price of private capital and the public project is viable if the following inequality holds: $\frac{B_1}{1.06} + \frac{B_2}{(1.06)^2} \geq \frac{1.16}{1.06}$. This criterion differs from that in which simply the private rate of return on capital is used as the discount rate.
15. This is based upon the following figures: $\rho = 1\%, \theta = 1$ and $g = 2.5\%$. $\theta = 1$ when preferences were logarithmic for example.
16. These policy changes came in response to previous reviews of the discounting literature (OXERA 2002).
17. This would not occur in the optimal Ramsey set up however since, for example, $f'(k) > 0$.
18. We thank an anonymous referee for alerting us to this approach. We assume in the discussion that g is primal.
19. That θ remains constant in the presence of positive or negative growth is akin to the commonly used modelling assumption that agents in the economy have constant relative risk aversion (CRRA). Clearly, this interpretation makes only partial sense in the deterministic case in which it is more sensible to talk of constant intertemporal substitution.

20. $G(.)$ is assumed to be continous and monotonic and is constant returns to scale.
21. G_{ψ} and $G_{\psi\psi}$ are the first and second derivatives of $G(.)$ with respect to ψ.
22. The total derivative of $\overline{D} = YG\left(\frac{\psi}{Y}\right)$ is: $0 = G(.) + YG_{\psi}(.)\left[\frac{\psi'}{Y} - \frac{\psi}{Y^2}\right]$. Rearranging this gives $\psi' = \frac{\psi}{Y} - \frac{G}{G_{\psi}}$.
23. Gravelle and Rees (2000) focus on health benefits for example.
24. Horowitz (2002) for example appears to confuse Weitzman (1994) and Fisher and Krutilla (1975) in this sense.
25. Preliminary work by Traeger (2004) shows that this widely held view may not be true where there is limited substitutability between environmental and produced goods in the utility function.
26. Similar ideas have been expressed in Sozou (1998) and Azfar (1999).
27. The yield curve shows the term structure of financial assets, that is, how the rate of return varies for assets with different maturities.
28. This is not crucial for this particular result to hold but is important for ease of exposition. The certainty equivalents could be defined to incorporate higher moments of the distribution of discount rates and to reflect risk aversion, but with a loss of tractability.
29. Note that the probability densities are assumed to be time invariant. This is not necessary for the result but as we shall see later, the nature of the probability distribution is of considerable importance for any estimated schedule of certainty equivalent discount rates.
30. It is the definition of the average certainty equivalent rate in equation (18) that has lead some commentators to describe Weitzman's CE as a restatement of Jenson's inequality since it effectively defines $r_{\mathrm{a}}^{\mathrm{CE}}$ as the harmonic mean of $\exp(-r_j t)$ (Newell and Pizer 2001). For example, if there are two possible interest rates with associated probabilities (r_1, r_2) and (p_1, p_2) respectively then

$$\exp(r_{\mathrm{a}}^{\mathrm{CE}} t) = \frac{\exp(r_1 t)\exp(r_2 t)}{p_1 \exp(r_2 t) + p_2 \exp(r_1 t)},$$

    which is a weighted harmonic mean of $\exp(r_1 t)$ and $\exp(r_2 t)$. This definition is strictly different to Weitzman's which is effectively a weighted arithmetic mean.
31. Another way to think about this is so say that, in the face of uncertain r, agents are unsure as to how to evaluate the opportunity cost of the project, and hence which discount factor to employ in determining the NPV. This is equivalent to stating that if agents desired a sure benefit of pounds 1 at time t, then given that they face an uncertain discount factor before the realisation of $\tilde{r}$, they are uncertain of the contribution they should make (Gollier 2002a). Agents must make some judgement of the discount factor and will use the certainty equivalent discount factor.
32. The last step is not entirely obvious, so we elaborate. Dropping the m subscript from $r_{\mathrm{m}}^{\mathrm{CE}}$, note that: $\dot{w}_j(t) = w_j(t)(\Sigma w_i(t) r_i - r_j) \;\; = w_j(t)(r^{\mathrm{CE}} - r_j)$, therefore $\frac{\mathrm{d}}{\mathrm{d}_t} r^{\mathrm{CE}} = \sum w_j(t)$ $(r^{\mathrm{CE}} r_j - r_j^2) = (r^{\mathrm{CE}})^2 - \sum w_j(t) r_j^2$. This term is equal to that obtained by multiplying out (21). That is, noting that $\Sigma w_j(t) = 1$ we get: $-\sum w_j(t)(r_j^2 + (r^{\mathrm{CE}})^2 \;\; -2 r_j r^{\mathrm{CE}}) = 2(r^{\mathrm{CE}})^2 - (r^{\mathrm{CE}})^2 - \sum w_j(t) r_j^2$ and we are done.
33. Gollier (2002a) provides an elegant proof of the following: $\lim_{t \to \infty} r_{\mathrm{a}}^{\mathrm{CE}} = r_{\mathrm{min}}$, i.e. for the average CER, by appeal to Pratts Theorem.
34. The tree economy describes a situation in which each individual is endowed with some productive capital, a tree, with uncertain exogenous growth rate, g, in the form of fruits. The fruits are perishable and therefore borrowing and lending occurs within periods with debts repaid by growth in future periods. In effect, therefore, capital is exogenous, and the interest rate that sustains the equilibrium is determined by individual characteristics that make up δ.
35. A discussion of this model is also found in Heal (1998).
36. Dasgupta (2001) attributes this critique to Kenneth Judd.

37. Note the mathematical equivalence of (34) with the average certainty equivalent rate defined in (18). π represents an intergenerational weight here rather than a probability in (18).
38. See, for instance, Thaler (1981), Cropper et al. (1994), Kirby (1997) and the review by Ainslie (1992).
39. Preliminary results from unfinished work by Benhabib et al. (2004) is also suggestive of alternative explanations.
40. The exponential discount function uses a discount rate of 3%. The Hyperbolic discount function has $\bar{\omega} = 0.03$ and $\xi = 0.1$. The Quasi-Hyperbolic function has $\beta = 0.5$ and $\zeta = 0.98$.
41. For comparison, standard discounting in discrete time is represented by the discount factors $\{1, \zeta, \zeta^2, \zeta^3, \ldots\}$ where $\zeta \approx e^{-\delta}$, the continuous-time analogue.
42. This has been clear since Strotz (1956). A formal statement of this proposition, including specification of the features of the discount function that generate time inconsistency, is provided by Hepburn (2005).
43. With the infinitely lived representative agent approach there is effectively only one agent, and thus one generation. The reference to current and future generations is therefore an intuitive interpretation of the long-run.

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Appendix A. The proof that $\lim_{t\to\infty} r_a^{CE} = r_{min}$

A rough sketch of the proof is as follows: $r_{\mathrm{a}}^{\mathrm{CE}}$ can be thought of as the certainty equivalent of a random pay-off, $\tilde{r}$, for an agent with a constant degree of absolute risk aversion t. In particular preferences are reflected by $E[u(r)] = E[-\exp(-\tilde{r}t)] = -\exp(-r_{\mathrm{a}}^{\mathrm{CE}}t)$. As risk aversion increases, i.e. t increase, it is well known that the certainty equivalent will decrease (Pratt 1964). Furthermore, as $t \rightarrow \infty, r_{\mathrm{a}}^{\mathrm{CE}}$ will tend to the lower bound of $\tilde{r} : r_{\min}$.

Appendix B. Explicit example of Weitzman's Certainty Equivalent Discount Rate

Suppose that there are two potential realisations of the discount rate (r_1, r_2) with associated probabilities (p_1, p_2). Using the definitions (16) and (19) we obtain the certainty equivalent discount factor and rate at time t:

$$A(t) = p_1\exp(-r_1 t) + p_2\exp(-r_2 t) = p_1 a_1(t) + p_2 a_2(t) = \sum p_j a_j(t)$$

$$r_{\mathrm{m}}^{\mathrm{CE}} = -\frac{\dot{A}(t)}{A(t)} = \frac{r_1 p_1 a_1(t) + r_2 p_2 a_2(t)}{p_1 a_1(t) + p_2 a_2(t)} = w_1(t) r_1 + w_2(t) r_2 = \sum w_j(t) r_j$$

where $w_1(t) = p_1 a_1/(p_1 a_1 + p_2 a_2)$ and $w_2(t) = p_2 a_2/(p_1 a_1 + p_2 a_2)$ and $\Sigma w_j(t) = 1$. This formula is used for $r_{\mathrm{m}}^{\mathrm{CE}}$ in Table 1. The formula for $r_{\mathrm{a}}^{\mathrm{CE}}$ is:

$$r_{\mathrm{a}}^{\mathrm{CE}} = -\frac{1}{t}\ln[p_1\exp(-r_1 t) + p_2\exp(-r_2 t)]$$

Using (21) and the fact that:

$$\dot{w}_j(t) = \frac{p_j a_j(t)}{\Sigma p_j a_j(t)} \cdot \frac{\Sigma r_i p_i a_i(t)}{\Sigma p_i a_i(t)} - \frac{r_j p_j a_j(t)}{\Sigma p_j a_j(t)} = w_j(t)(r_{\mathrm{m}}^{\mathrm{CE}} - r_j)$$

the derivative of $r_{\mathrm{m}}^{\mathrm{CE}}$ with respect to time then becomes:

$$\frac{\mathrm{d}}{\mathrm{d}t} r_{\mathrm{m}}^{\mathrm{CE}} = -[w_1(r_{\mathrm{m}}^{\mathrm{CE}} - r_1) r_1 + w_2(r_{\mathrm{m}}^{\mathrm{CE}} - r_2) r_2] = -\sum w_j(t)(r_{\mathrm{m}}^{\mathrm{CE}} - r_j)^2$$

Appendix C. The Lucas Tree model

Gollier (2001, p. 250) explains concisely the approach taken. The maximisation problem is a dynamic programme in which the equilibrium value function for individuals is:

$$v_t(y,b) = \max_{c_t}\{u(c_t) + \beta E v_{t+1}(y(1+g_{t+1}), (1+r_t)(c_t+b-y))[u(\tilde{y}_\tau)|\tilde{y}_t = y]\}$$

where y is income (size of crop from trees) and b is repayment of debt (borrowed fruit). Commodities are assumed to be perishable and borrowing and lending occurs across time measured by the quantity $c_t + b - y$. Income y is exogenous but grows at the uncertain rate $\tilde{g}_t$, which is assumed to be *i.i.d* across time. The first order conditions for maximisation are:

$$u'(c_t) = -\beta E v'_{t+1}(y(1+g_{t+1}), (1+r_t)(c_t+b-y)) \tag{37}$$

Since all individuals are identical, the equilibrium in this economy is autarkic such that at time t the individual pays back and debt, b, and consumes such that $c = y - b$. There is no borrowing, hence $c_t + b - y = 0$. This means that the equilibrium value function at time t is:

$$v_t(y,b) = \max_{c_t}\left\{u(c_t) + \sum_{\tau=t+1}^{T} \beta^{\tau-1} E[u(\tilde{y}_\tau)|\tilde{y}_t = y]\right\}$$

Hence the derivative of the value function with respect to the state variable, b, is $v'_t = -u'(y-b), v'_{t+1} = -u'(y(1+\tilde{g}_{t+1}))$. Using these and rearranging (37) we obtain:

$$1 + r(c_t) = \frac{u'(c_t)}{\beta E[u'(c_t(1+\tilde{g}_{t+1}))]}$$

Equation (28) can be found by first defining the *precautionary equivalent* growth rate $\hat{g}_{t+1}$ as the certain growth rate that yields the same interest rate as in equation (27), that is:

$$\hat{g}_{t+1} : E[u'(y(1+\tilde{g}_{t+1}))] = u'(y(1+\hat{g}_{t+1}))$$

Taking second order Taylor series expansions of both sides yields:

$$\hat{g}_{t+1} = E\tilde{g}_{t+1} - \frac{1}{2}\text{var}(\tilde{g}_{t+1})\frac{u'''}{u''}y$$

Inserting this into the Ramsey rule: $r = \rho + \theta g_{t+1}$ gives us (28).

Appendix D. Empirical Specification

See Newell and Pizer (2003) for their empirical specification. The State Space model employed here is as follows:

$$r_t = c_1 + \alpha_t r_{t-1} + e_t$$

$$\alpha_t = c_2 \alpha_{t-1} + u_t$$

where u_t and e_t are vectors of serially independent zero-mean normal disturbances. In other words, we model uncertainty of the interest rate as an $AR(1)$ process with $AR(1)$ coefficients. Details of this and other specifications can be found in Groom et al. (2004).