Discounting and the Social Cost of Carbon

Jiehan Guo

MSc Thesis for Environmental Change and Management

University of Oxford

16 August 2004

Supervisor: Cameron Hepburn

Acknowledgements

I'm much indebted to my supervisor, Dr Cameron Hepburn for his supervision and encouragement, which is far beyond the call of duty. I'm also grateful to the Stockholm Environment Institute Oxford Office and Dr Thomas Downing for providing me the opportunity to work in the DEFRA project of Social Cost of Carbon and providing scholarship for my work. Professor Richard Tol from Hamburg University kindly offered invaluable help to enable me to understand the FUND model during and after our visit to Hamburg. I would also like to thank my group-mates: David Anthoff, Megan Ceronsky and Ada Li for being such nice work-mates, particularly David, who has offered large amount of time and energy to help me with programming. Xin also deserves my gratitude for giving me advice on revising this dissertation and keeping me going generally. Finally, I'm forever grateful for my parents' love and support during my study.

Table of Contents

Abstract

This dissertation carries out a sophisticated sensitivity study on the impacts of declining discount rates on the social cost of carbon (SCC), an important number in the economic appraisal of climate change policies. Five declining discounting schemes are successfully implemented in the FUND 2.8 model. Combined with different assumptions of the pure rate of time preference, these five DDR schemes produce 10 estimates of the SCC number. Among them, the Gollier heterogeneous discounting scheme is, to the best of our knowledge, implemented for the first time in the literature. Without equity weighting, the percentages increase of SCC values ranges from 10% to 4100% and the value of SCC ranges from -£1.4/tC to £128/tC. Although this uncertainty range is large, most discounting schemes and combinations don't push up the number to the high level suggested by UK DEFRA (2002). The novel implementation of the Gollier heterogeneous discounting even suggests the possibility of negative SCC, although it also has to do with the damage profile in FUND. One of the major policy implications is that at the higher end of the values of SCC found here (although not all of them), many climate change related policies — such as the Kyoto Protocol — have no trouble passing a cost-benefit analysis.

List of Abbreviations

BAU	Business as Usual
CBA	Cost-benefit Analysis
CEDF	Certainty-Equivalent Discount Factor
CEDR	Certainty-Equivalent Discount Rate
DEFRA	UK Department of Environment, Food and Rural Affairs
DDR	Declining Discount Rates
FUND	The Climate Framework for Uncertainty, Negotiation and Distribution
GDP	Gross Domestic Products
IAM	Integrated Assessment Models
IMF	International Monetary Fund
IPCC	Intergovernmental Panel on Climate Change
MAC	Marginal Abatement Cost
MD	Marginal Damage
NPV	Net Present Value
PRTP	Pure Rate of Time Preference
SCC	Social Cost of Carbon
SDR	Social Discount Rate
SOC	Social Opportunity Cost of Capital
SRTP	Social Rate of Time Preference

List of Tables

List of Figures

1. Introduction

The recent film ‘The Day After Tomorrow’ triggered further criticisms on the US pullout of the Kyoto Protocol. After the US withdrew from the Kyoto Protocol in 2001, Russia's 17 percent emission quota leaves it with the casting vote in the ratification of the protocol. Yet Russia has vacillated over the decision to voluntarily limit carbon dioxide emissions. One of its major considerations is the costs and benefits of implementing the protocol. According to a recent report by the Russian Academy of Sciences (The Russia Journal, 2004), ratifying the protocol will ‘seriously harm’ the Russian economy yet the benefit to Russia is ‘scientifically unfounded’. The same argument has been used as a reason for the pullout of the US (Pearce, 2003). Although some may object to such cost benefit analysis on the basis that the potential damage is so large that action must be taken irrespective of the costs, it can be seen that comparing costs and benefits of controlling global warming is an important step in decision making for individual countries and the world as a whole.

The ‘social cost of carbon’ (SCC) is an important element in such cost-benefit analysis. Also called the ‘shadow price’ of carbon, SCC is the monetary indicator of the global incremental damage done by emitting greenhouse gases today, carbon dioxide being the major one. In cost-benefit analysis of projects to control greenhouse gas emissions, the SCC stands for the benefit of the project, in terms of ‘avoided damage’. Thus, the larger the number of SCC, the more control of global warming is justified.

Integrated Assessment Models (IAMs) combining the scientific prediction of global warming and the socio-economic analysis of the impacts have been used to estimate the

SCC since early 1990s. In these models, impacts at different times in the future are estimated, discounted back to present value and added up to a single number. Therefore, the choice of discount rates and discounting schemes has a significant influence on the final number of SCC and is one major source of uncertainties in the estimates (Tol, 2004). Traditionally, constant discount rates have been used in discounting, meaning that damages far into the future will count very little at present. Recent economics literature suggests that a time varying, particularly a declining discount rate might be more suitable for the estimate of SCC (Pearce, 2003).

Based on this, the aim of this dissertation is to carry out a reasonably sophisticated sensitivity study on time-varying discounting schemes and their impacts on the SCC. Such a study will provide more information to policies on climate change, both in terms of the range of uncertainties on SCC, and practical implications for implementing DDR in policy appraisals.

Declining discount rates based on different rationales are the focus of the dissertation. The sensitivity study is mainly based on a well-developed Integrated Assessment Model called FUND (Climate Framework for Uncertainty, Negotiation and Distribution), developed in the late 1990s by Professor Richard Tol in Hamburg University. Different discounting schemes are applied and the outcomes are then compared, followed by discussions on policy implications.

The dissertation is to proceed as follows. Chapter 2 gives an overview of the social cost of carbon. Chapter 3 introduces theories on social discounting and declining discount rates. Chapter 4 discusses the methods used to carry out the sensitivity studies with the

FUND 2.8 model. Chapter 5 presents the results of the implementation and discusses policy implications of these results. Chapter 6 concludes.

2. Overview on the social cost of carbon (SCC)

2.1 Rationales for the SCC

Before proceeding to discuss exact estimates of the SCC, it is necessary to justify such a practice. The first justification is that SCC helps to determine the economically 'efficient' level of climate change abatement. Like many forms of environmental pollution, the optimal level of greenhouse gas pollution is not zero because it entails costs to reduce such pollution. Presumably, we will continue to reduce pollution as long as the benefits of doing so exceed costs. But when a further incremental ('marginal' in economics terms) reduction in pollution implies more costs than benefits, it is sensible not to go any further and stay at that 'optimal' level of pollution. This is the process of cost-benefit analysis in economics and SCC represents the benefits of controlling climate change, or interpreted as avoided damages.

It should be noted that this function of SCC does not require it to be precise. Acting on some reasonable estimates is arguably better than no estimates because monetary valuation is most likely inevitable. As Thomas (1963) pointed out, 'the setting of any quality criterion or standard relating to health and wellbeing inevitably entails making an implicit estimate of cost/benefit ratios based on whatever data or other factors available.' In other words, where policy costs money, monetary valuation is inevitable, whether implicitly or explicitly.

This leads to the second rationale for estimating the SCC- if monetary valuation is inevitable, it is better done explicitly than implicitly. Implicit use of the SCC will lead to policy inconsistencies, i.e. different numbers of SCC are used in different policies and such inconsistencies further cause efficiency loss. The same argument could be used to respond to the substantial literature opposing monetization of damages, as explained in more details in Pearce (2003). The main point is that however well-intentioned the criticisms are, some form of comparison of costs and benefits is unavoidable.

2.2 The process for estimating the SCC

The SCC is estimated through models of varying degrees of sophistication. These models usually stick a socio-economic 'back-end' to climate change predictions. The number estimate of SCC can be obtained from two approaches: the marginal cost (MC) approach and the cost-benefit approach. The marginal cost approach is generally done in the following steps (see also Figure 2.1).

1. The world is divided into *i* regions and an emission scenario is selected as the predicted path of future emission.
2. Then damages for each period and each region are estimated. This is done through several linkages, from emissions to concentration of CO_2, from concentration to temperature rise and from temperature rise to damages. The first two linkages are the task of the climate change part of the model while the last part relies on the socio-economic part of the model. These predictions are often obtained through a bottom-up approach estimating sectoral damages. When sea-level-related damages are of concern, the linkage process may also include an intermediary linkage from temperature rise to sea level rise.

3. An extra ton of CO_2 at present is added to the emission scenario and the new damages per region per period are estimated.

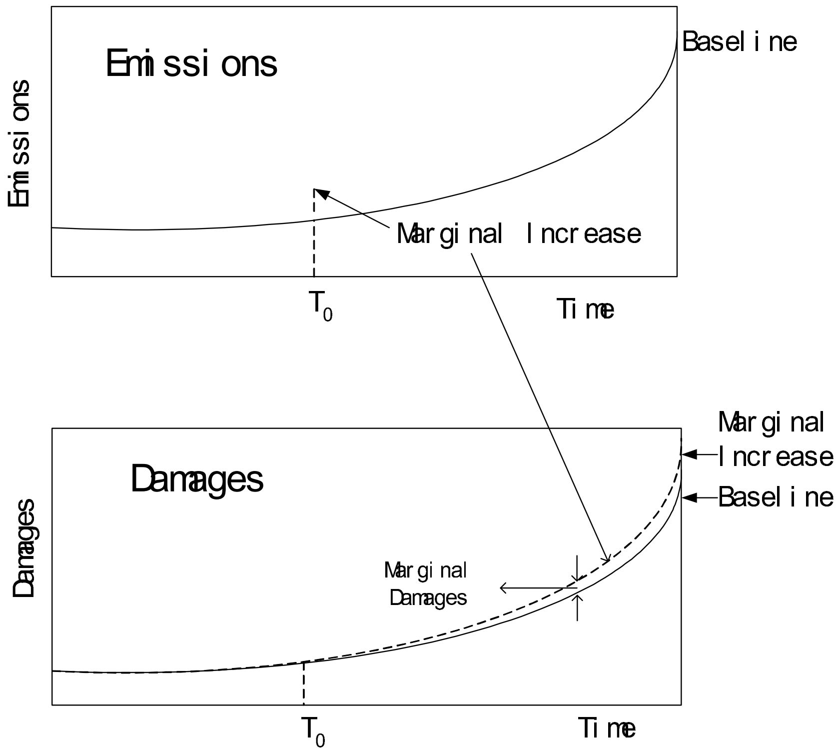
4. The differences between the damages in 2. and 3. are obtained, expressed as D_{it}, which are still in the form of per region per year.

5. D_{it}'s are discounted back to present values using the social discount rate, s: $V_i = \sum_0^T D_{it}(1+s)^{-t}$, where V_i is the present value of damages in region i.

6. The present value damages of each region V_i's are added together, with equity weighting if necessary. Equity weighting is based on the rationale that \$1 damage to a poor person should be placed on more weights than the same amount of damage to a rich person. Therefore, the impacts of climate change on developing countries should be valued more than those on developed rich countries. How such a weight is chosen differs from models to models. But the weight is generally linked to the average income of the region. For example, in one weighting scheme, the total world damage can be expressed as: $V_{world} = \sum_i V_i \cdot \left[\frac{\overline{Y}}{Y_i}\right]^{\varepsilon}$, where $\overline{Y}$ is the world average income and Y_i is the average income in region i. ε is the elasticity of marginal income, a measure of 'inequality aversion'. For more details, see Anthoff (2004)

V_{world} is then the number of SCC, in the form of monetary value for an extra ton of CO_2 emitted today.

Figure 2.1 The Marginal Cost Approach

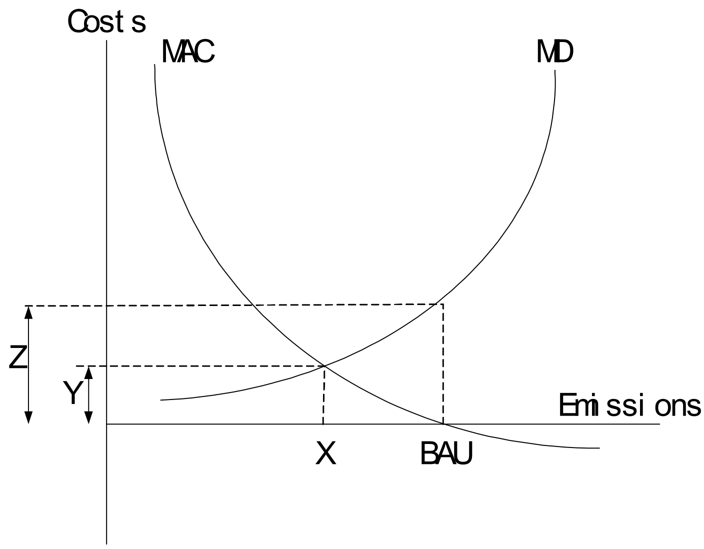


Source: Adapted from Clarkson and Deyes (2002)

Besides the MC approach, the 'cost-benefit approach' is also used. This approach aims to calculate the socially optimum level of emissions. This level is obtained at the intersection of the marginal damage (MD) and the marginal abatement cost (MAC) curves. The MAC curve comes from the modelers' knowledge of the abatement cost of reducing current emissions by an extra ton of CO_2. The SCC is then defined as the pollution tax required to keep emissions at the optimal level. This process can be shown in Figure 2.2 below.

In Figure 2.2, it is assumed that the marginal costs/damages to the private sector are equal to zero at all levels of emissions, i.e. the MD curve for private sector is the horizontal axis. Therefore, under the business-as-usual (BAU) scenario, the private sector will emit at a level where their MD curves (the horizontal axis) intersects with the MAC curve (the BAU point in the figure). At this level, the social marginal damage equals Z in the figure, which is also the result of the marginal cost (MC) approach. To the society, the optimal level of emission is where MAC equals social MD, point 'X' in the figure. The carbon tax needed to keep emission at this optimum level is the distance Y in the figure, which is the difference between social and private damages. Therefore, Y is the SCC from the cost-benefit approach. If we assume that the BAU emission is usually larger than the socially optimal level, the SCC from the MC approach is generally bigger than that from the cost-benefit approach (distance Z as opposed to Y in Figure 2).

Figure 2.2 The Cost-Benefit Approach



Source: Clarkson and Deyes (2002)

2.3 Current estimates of the SCC

Variations between different estimates are considerable. But the range from the following significant reviews is much smaller.

- In 1996, the IPCC Working Group III published a review on the first generation models (Pearce et al., 1996). The range of SCC in the review is \$5- \$125 per ton of carbon (in 1990 prices, or \$6 - \$ 160 /tC in 2000 prices). It was also estimated that this range was to increase to \$7- \$ 154/tC (in 1990 prices) for the period 2001-2010 (because the MD of carbon tends to increase with the level of atmospheric concentration of CO_2). Some of the pioneering studies in SCC modeling include Nordhaus (1990, 1994), Cline (1992) and Fankhauser (1995), etc.

- In 2002, the UK Government Economic Service (GES) and DEFRA (Department for Environment, Food and Rural Affairs) published a review based on 8 major studies (Clarkson and Deyes, 2002). The recommended SCC was approximately £70 /tC (\$101.5 /tC)[1]in 2000 prices with equity weighting, with a range of £35- £140 /tC (\$51- \$203 /tC). Such an estimate was said to be based on Eyre et al. (1999), a 'most sophisticated' estimate. And it was suggested that this figure should rise at £1/tC for each subsequent year.
- Pearce (2003) reviews the studies in the Clarkson and Deyes (2002) and four more peer-reviewed studies. The range of SCC from the paper is £3-6 /tC (\$4.35-8.7/tC) without equity weighting, while equity weighting raises the range to £3-15/tC (\$4.35-21/tC). Time varying discount rate will raise the range further to £4-27 /tC (\$5.8-39/tC). This much lower estimate is attributed to the consideration of adaptive behaviour, which the author admits as likely to be 'too optimistic'.
- In a recent working paper, Tol (2004) gathers 103 estimates from 28 published studies to form a probability density function. The mode is \$2/tC, the median \$14/tC,

---

[1] Applying a conversion rate of 1.45 US \$ per UK£, the same applies to the rest of the conversions

the mean \$93/tC, and the 95 percentile \$350/tC. The conclusion was that the marginal damage costs of carbon dioxide emissions are unlikely to exceed \$50/tC, under standard assumptions about discounting and aggregation, and probably much smaller.

2.4 Uncertainties in SCC estimates

Many uncertainties lead to the wide range of estimates of SCC and these uncertainties can be grouped into three categories:

- Scientific uncertainties: uncertainties about present and future emissions and the impacts under different emission scenarios. More detailed discussions of these uncertainties can be found in the Third Assessment Report (TAR) by IPCC (IPCC, 2001). A particular area of concern is the risk of low-probability catastrophes (such as the melting of the West-Antarctic ice sheet and the shut-down of the thermohaline circulation). Not many current studies on SCC have taken these risks into account.
- Economic and policy uncertainties: Estimating the monetary values for non-market impacts (impacts for which a market price doesn't exist) poses a challenge to economists. For example, it is hard to place a value on the loss of biodiversity. It is also difficult to know how policies will respond to future impacts.
- Ethical value judgments: The aggregation of values across time and regions depend on the choice of a discounting scheme and equity weighting scheme respectively. This choice is largely a matter of ethical judgment and is, and 'will likely remain, an unresolved question in economics' (IPCC, 1996). The choice of discount rates has

been found to be a major driver for uncertainties (Tol, 2004).

2.5 Chapter Summary

This chapter first justifies the practice of estimating SCC with two rationales: the need for an optimal level of climate change control and the need to do CBA explicitly to avoid policy inconsistencies. The chapter goes on to explain the two major approaches to estimate the SCC: the marginal cost and the cost-benefit approach. Recent literatures on the SCC and the causes of uncertainties in SCC are generalized. The next chapter will deal with theories on one of these uncertainties-discounting.

3. Theories on social discounting

3.1 Discounting and cost-benefit analysis

Discounting is used to compare costs and benefits that occur at different points of time. In other words, £1 today is not regarded the same as £1 tomorrow because generally speaking, people prefer to receive goods and services sooner rather than later. Also called the 'discounted utility' approach, the practice of discounting was first set up by Samuelson (1937). Although Samuelson himself never intended that the discounted utility model be taken as the norm or the description of actual behaviour in terms of inter-temporal decision-making (Frederick et al.2002), it has been widely adopted since its introduction (OXERA, 2002).

Not only individuals use discounting to make inter-temporal choices, the society also relies on it to make informed investment decisions. Like other investments, investment in the environment involves incurring costs today for benefits in the future. Whether such an investment is efficient is determined by social cost-benefit analysis (CBA). In a competitive economy (where there is no market failure), the socially efficient level of investment is attained by investing in projects where the net present value (NPV), determined by discounting costs and benefits at the social discount rate (SDR) over the time horizon, is greater than zero. The conventional process of CBA can be illustrated as below.

i. Suppose a project spans over a time horizon of t, and the benefits and costs at each period t are denoted as B_t and C_t respectively. Before they can be compared, they are discounted back to present value (*PV*) using the social discount rate (*s*):

$$PV(B_t) = \frac{B_t}{(1+s)^t}, PV(C_t) = \frac{C_t}{(1+s)^t} \tag{3.1}$$

ii. Then the present values of benefits are added up:

$$PV(B) = \sum_t PV(B_t), PV(C) = \sum_t PV(C_t) \tag{3.2}$$

where *PV(B)* and *PV(C)* are the sum of the present values of benefits and costs.

iii. The net present value (*NPV*) is calculated:

$$NPV = PV(B) - PV(C) \tag{3.3}$$

iv. The efficiency criteria is whether *NPV*>0. If *NPV*>0, the project is said to have passed the cost-benefit analysis and is efficient.

Although efficiency is not the only criteria on investment decisions (for example, equity is another concern, which will be discussed later), it is among the most important ones. And it follows from the above description that the choice of the social discount rate *s*, has a big influence on the final outcome of CBA.

Besides the *discount rate*, another important parameter is the *discount factor*. Equation (3.1) can be transformed as:

$$PV(B_t) = \frac{1}{(1+s)^t} \cdot B_t \tag{3.4}$$

The first term on the right of (3.4) is called the **discount factor**, that is:

$$DF_t = \frac{1}{(1+s)^t} \tag{3.5}$$

where DF_t refers to the discount factor at time *t*. In the conventional formulation, *s* is constant and does not vary with time. Under this assumption, the link between the discount factor and the discount rate can be shown as[2]:

$$s = \frac{DF_{t+1}}{DF_t} - 1 \tag{3.6}$$

Equations (3.4) and (3.5) lead to an important distinction between the discount factor and the discount rate. It is the *discount factor* that is the actual weight placed on values at different points in time, not the *discount rate*. A crucial implication of this distinction is that any uncertainty about the weight to be given to future interests will be uncertainty about the *discount factor*.

3.2 Candidates for the social discount rate

Issues relating to social discounting have occupied an important place in economics debates. These debates mainly revolve around two questions: what discount rate to be used as the social discount rate (SDR) to reflect society's preferences and how to derive this rate. This section focuses on the first question and the next section on the second.

There are mainly two candidates for the SDR. One is called the 'social rate of time preference (SRTP)' and the other is the 'social opportunity cost of capital (SOC)'. The SRTP is the 'rate of fall in the social value of *consumption* by the public, as opposed to public sector income' (Pearce and Ulph, 1999). It is also known as the 'consumption rate of interest' while the SOC is also called the 'social rate of return on investment'. SOC is identified with the real rate of (social) return earned on a marginal project in the private sector. The idea is that if capital investment is diverted from the private sector to the

[2] More formally, the discount rate is the rate of change of the discount factor, expressed as: $s_t = \frac{\partial DF_t / \partial t}{DF_t}$

public project (through means such as taxation), the return of that project should at least be the same as the return on a private project. Otherwise it will make more sense not to do so.

The main difference between these two discount rates is *what* is to be discounted. The SRTP is discounting the public *consumption* while the SOC is discounting the *capital investment flows* (or public sector *income*). In an economy without any distortions (such as taxes, market power, or externalities of any kind), these two rates are the same. But in reality they are rarely equal. Hence there is a problem of choosing between them.

Since these two discount rates represent different numeraires, choosing any one of them would mean that there is a need to convert either consumption or investment into the other. In other words, there is a need to reconcile these two rates. A widely accepted approach in the literature (e.g. Cline, 1992) is a revised benefit-cost decision rule in which both rates appear. The procedure is to look at each unit of investment cost and classify the sources of the cost according to whether they come from consumption or investment. If the cost comes from investment, it is converted to consumption equivalent units through 'the shadow price of capital', v (Lind, 1982). The conversion factor v is influenced by the SOC rate. Then the resulting consumption equivalent flows can be discounted at the SRTP, the 'fundamental' discount rate in this approach. The same process should be applied to the benefits arising from the project.

It seems that if the 'shadow price of capital' can be determined, this revised approach settles the problem of choosing a discount rate. But the real difficulty is deciding what portion of the investment comes from consumption and what portion comes from investment (Pearce and Ulph, 1999). Due to the complexity of this method and the fact that SRTP and SOC are very close to each other, the SRTP is more often used. For example, the UK Green Book (2003) recommends SRTP to be used as the social discount rate. Later discussions in this dissertation also focus on SRTP.

3.3 Derivation of the social discount rate (SRTP)

There are several ways of deriving the SRTP. One way is through empirical studies. In these studies, what is observed is a discount factor- the weight placed on different points of time. Thus a discount rate can be inferred from these weights. For example, Warner and Pleeter (2001) reported personal discount rates based on evidence from the US military downsizing programs in the early 1990's. Frederick et al. (2002) makes a comprehensive survey on relevant studies. To the extent that the social discount rates represent the aggregate of individual behaviour, the personal discount rates estimated from these empirical studies also reflect the SDR. One characteristic of these studies is that there is substantial variability between different estimates, because discount rates may vary with context.

Another usual approach is to decompose SRTP into several parameters and estimate the parameters separately. The Ramsey Equation (Ramsey, 1928) is often used in this instance. The Ramsey Equation gives the social rate of time preference (SRTP) as:

$$s = \rho + \mu \cdot g \tag{3.7}$$

where s is the SRTP; ρ is called 'pure rate of time preference'(PRTP), and is the rate at which individuals discount future *utility* (well-being, welfare), also known as the 'utility discount rate'; μ is the income elasticity of the marginal utility and g is the growth rate of per capita real consumption. Pearce and Ulph (1999) further decompose the first term in the right of the equation into:

$$\rho = \delta - L \tag{3.8}$$

where δ is now the 'true' utility discount rate, a rate independent of any risks to life; and L is the rate of growth of life chances. If life chances gets worse through time (L gets smaller), then the ρ will be higher, meaning that people will prefer the present even more to the future. Each component in (3.7) and (3.8) will be explained below.

The 'pure' rate of time preference δ reflects individuals' preference for consumption now, rather than later, with consumption per capita unchanged over time. It is also called the 'utility discount rate'. There are controversies on whether δ should be equal to zero. Implications of this will be discussed in next section.

Changing life chances, L, refers to the likelihood of catastrophes or events that reduce the chances of life. Combining these two elements, ρ in (3.7) is estimated at around 1.5% in the literature.

The income elasticity of marginal utility, μ, measures the changing level of happiness (utility) we get from an extra unit of income as our income level increases. The literature suggests that this value is around 1 (e.g. Cowell and Gardiner, 1999; OXERA, 2002). This implies that an extra pound to a generation that has twice the consumption of the current one will only bring half as much happiness to that generation.

The growth rate of per capita consumption, g varies from country to country. Predictions are usually made on the observation of past trends.

It should be noted that since the actual discount rate may vary greatly with context, such an attempt to deconstruct the discount rate might not be appropriate under some circumstances. Nevertheless, the Ramsey equation is one of the most widely adopted approach in social discounting.

3.4 The ethics of discounting

As mentioned in 3.1, equity is another criterion in inter-temporal choices. Some believers in equity would argue that while it is plausible to discount consumption, it is unreasonable to discount future wellbeing (e.g. Broome, 1992). That means the SRTP in can be positive but the utility discount rate (PRTP) has to be zero. Such a proposition is supported by many economists (including Ramsey himself), philosophers and environmentalists.

The main argument is that a positive PRTP introduces unfairness for future generations, who have no say in our present decision on investment. However, zero discounting has its own problem. As was shown by Olson and Bailey (1981) and Koopmans (1960), as long as the interest rate is positive and the discount rate is zero, there will always be justifications for the current generations to reduce consumptions to subsistence level, in the name of increasing future generations' consumption. The logical implication of such a situation is that the poorest people today will sacrifice their wellbeing for a better-off future generation. This is contrary to the Rawlsian rule of intergeneration equity (Rawls, 1972), in which society should aim to maximize the wellbeing of the poorest among all generations.

It seems that neither a positive or a zero PRTP is perfect. The tension between these two methods perhaps reveals the fact that loading the task of both intergenerational *efficiency* and *equity* on just one parameter, the discount rate, is itself unwise (OXERA, 2002).

A second ethical problem with conventional positive discounting arises in investments over a long time horizon, such as those related to climate change. The discount *rate* in conventional discounting remains constant over time and the discount *factor* d is expressed as: $d= (1+s)^{-t}$ for discrete time. But when time intervals become very small and thus continuous, the discount factor is expressed as $d= exp(-st)$ for continuous time, where s is the discount *rate*. Therefore, the discount *factor* is falling exponentially with time. The impact of the conventional discounting can be illustrated by a simple example. Consider being paid \$100 at some point in the distant future. At a discount rate of 3%, the 'present value' of \$100 is worth: \$22.8 at year 50 (t = 50 in equation (3.1)); \$5.2 at year 100; and \$0.27 at year 200.

It can be seen that conventional discounting can reduce the value in 200 years from now to almost nothing. The implication is that we don't care about what happens in the far future. While this might be true for some people, it is unlikely to be a satisfactory basis for policy making. As Weitzman (1998) states, 'to think about the distant future in terms of standard discounting is to have an uneasy intuitive feeling that something is wrong, somewhere'. This is also contrary to the widely supported goal of sustainable development, which calls for concerns for welfare of the future over long time horizons. One solution to the problem of long-term discounting is time-varying, especially declining discount rates.

3.5 Declining discount rates

The equity problem with conventional discounting prompted economists to come up with something else. Time-varying, particularly declining discount rate is one answer. It means that we continue to discount using the same formula (i.e. (3.1) above), but the discount rate is declining over time, so that the weight placed on the far-distant future is increased (the discount factor *DF* in (3.5) increases with the decrease of *s*) compared with conventional discounting. Far from being an *ad hoc* solution, declining discount rates (DDR) are supported by ample evidence both empirically and theoretically.

3.5.1 Hyperbolic discounting

The first evidence comes from observation of people's behaviour. There is evidence that individuals' discount rates decline with time, following a hyperbolic path (e.g. Federick et al., 2002). To illustrate, if we were faced with two choices: i) postponing our consumption for 1 year from now and ii) deferring an equal amount of consumption for 1 year from year 50 to year 51, we are likely to respond differently. While postponing the consumption right now might mean a lot, postponing it for an equal amount of time in 50 years from now might not. In other words, the weight we place on an extra year in the future is declining with time.

Hyperbolic discounting means that the discount *factor* declines as a hyperbolic function of time. A general function proposed by Loewenstein and Prelec (1992) is as follows:

$$d_t = \frac{1}{(1+kt)^{h/k}} \qquad (3.9)$$

where d_t is the discount *factor*. The parameter k measures the deviation of the hyperbolic discounting function from the standard exponential model. As k approaches zero, d_t approaches the exponential function. Another popular form of hyperbolic discounting is (Cropper et al, 1992):

$$w_t = \frac{1}{t^h} \qquad (3.10)$$

If social decisions should reflect individuals' choices, then hyperbolic discounting should be considered. But hyperbolic discounting is not without problems. As with almost all other forms of time-varying discount rates, time-inconsistency is one problem, which will be discussed in more details in 3.5.5. Another problem with hyperbolic discounting is that the parameters in (3.9) that are measured empirically imply very large initial discount rates, sometimes as high as 30-40%, although the rates are falling rapidly afterwards (For further explanations, see Appendix 1). Such high initial rates seem not reasonable for practical social decision making.

3.5.2 DDR based on uncertainties

The second well-supported rationale for declining discount rates is uncertainties, both about the present and about the future.

In his two influential papers, Weitzman (1998, 2001) set up a framework for DDR with recommendation on the specific path of decline, called 'Gamma Discounting'. The key point is that if we are currently uncertain about the weights to attach on the future, we should use a weighted discount *factor*. This weighted discount factor is also called the

'Certainty Equivalent Discount Factor' (CEDF). And once the weights placed on different discount factors are settled, they last forever. The result of this is that the actual discount rate (the 'Certainty Equivalent Discount Rate', CEDR that is derived from the discount factor) is declining over time. This can be illustrated by the example below. Suppose there are three possible discount rates to be used: 1%, 3% and 5%, each with an equal probability of 1/3. Then the certainty equivalent discount factors ($CEDF_t$) and the implicit discount rates ($CEDR_t$) over the time horizon should be calculated as:

$$\frac{1}{(1+CEDR_t)^t} = \frac{1}{3} \times \left[\frac{1}{(1+0.01)^t} + \frac{1}{(1+0.03)^t} + \frac{1}{(1+0.05)^t} \right] \tag{3.11}$$

It means that each possible discount *factor* is falling exponentially, in the same way as in conventional constant discounting, while the Certainty Equivalent Discount Rate is declining. The results of (3.11) are shown in Table 3.1 below.

Weitzman further shows that the lower limit of the CEDR is the lowest possible discount rate (1% in the example above). The reason for the decline of CEDR is that exponential discounting actually reduces the weight of higher discount rates as time goes by because their respective discount factors fall faster. In Table 3.1 above, it can be seen that the discount factors based on 1% becomes more dominant in the weighted sum as time goes along. Hence the discount rate falls closer to 1%.

Weitzman shows a probability distribution of possible discount rates- the gamma distribution- by conducting an email survey on economists in the world on the discount rate to be used in long-term discounting. That's why his proposition is also called 'gamma discounting'. Under this assumption, the function of discount rates is proposed as:

$$R(t) = \frac{\mu}{1 + t\sigma^2 / \mu} \qquad (3.12)$$

where μ and σ are the mean and standard deviation of the gamma distribution respectively.

Table 3.1 Numerical Illustration of Weitzman's Declining Discount Rate

Discount Rates	*Discount Factors in Period t*				
	10	50	100	200	500
1% (p_1=1/3)	0.905	0.608	0.370	0.137	0.007
3% (p_2=1/3)	0.744	0.228	0.052	0.003	0.000
5% (p_3=1/3)	0.614	0.087	0.008	0.000	0.000
Certainty Equivalent Discount Factor ($CEDF_t$)	0.754	0.308	0.143	0.046	0.002
Certainty Equivalent Discount Rate[3] ($CEDR_t$)	2.86%	2.38%	1.96%	1.55%	1.22%

Source: Own Calculation

Another uncertainty that leads to DDR is uncertainty about current and future growth rate of consumption and the economy (recall the parameters $\mu \cdot g$ in the Ramsey equation (3.7)), which was developed by Gollier (2002a, 2002b) in a different framework from Weitzman's.

Gollier's conclusion depends on the assumption of future economic growth and on μ, the index of risk aversion (see Appendix 2 for mathematical details). If there is no risk of recession both in the near and distant future, the discount rate is declining only if individuals display decreasing aversion to risk as wealth increases (i.e. μ decreases with

---

[3] This is actually the 'average' certainty equivalent discount rate. The 'marginal' CEDR, which is calculated slightly differently, but on the same principle, also falls with time, and more quickly.

income). There is some evidence that wealthier people have a higher appetite for risk: the share of the wealth invested in risky assets increases with income in both developed and developing countries (Ogaki and Zhang, 2000; Gollier, 2002a).

When there is a risk of recession, more restrictions are needed to get declining discount rates. An important restriction is whether individuals are 'prudent'. Prudence is introduced by Kimball (1990) to measure people's propensity to increase savings when faced with future risks. Put another way, if an individual is prudent, s/he is likely to 'save up for the rainy days'. Mathematically, the index of prudence is related to the 2nd and 3rd derivative of the utility function in consumption. If individuals both show decreasing relative risk aversion and increasing prudence, the discount rate is declining with time when risks exist only in the long term. When risks exist both in the short and long run, the situation becomes even more complex and further restrictions up to the 5th derivative of the utility function are necessary to generate a DDR. Such restrictions were recognized by Gollier himself as not possible to test in the near future. Even so, Gollier's theory provides significant addition to the literature of DDR.

Although developed in different theoretical frameworks, there is one thing similar to both Weitzman's and Gollier's theories- we should use the expected value of all possible discount factors.

3.5.3 DDR based on intergenerational equity

While the rationale for DDRs in the last section is based on uncertainty, the following contributions from Chichilnisky (1997), and Li and Löfgren (2000) justify DDRs on the grounds of intergenerational equity and sustainability. In the following descriptions, mathematical details are omitted.

Chichilnisky (1997) introduces two axioms for sustainable development that in combination require that neither the present nor the future should play a dictatorial role in society's choices over time. The implications of her formula of discounting is that the future will be discounted in a conventional manner in the near future, but after a point- the so-called 'switching date'- remaining effects will not be discounted (i.e. at a zero rate). (see Appendix 3)

Li and Löfgren (2000) treat the future in a different way. In recognition of the importance of future sustainability, they suppose there are two different individuals ('utility streams') in the society, a utilitarian and a conservationist. They have identical utility function, with consumption and resource stock as arguments. The difference between them is that they have different PRTP (ρ in (3.7)). The overall societal objective is to maximize a weighted sum of utility for both of them. The result of this weighting practice is similar to that of Weitzman's discounting- the individual with lower discount rate is given dominant weight as time goes by and the collective discount rate is declining. The technical difficulty with applying this approach is choosing a discount rate for both person and the weight to place on them (see Appendix 4).

3.5.4 DDR based on heterogeneous time preferences

Following Li and Löfgren's (2000) assumption about different PRTPs for two types of agents, it's reasonable to assume that preferences within a group of many people may be heterogeneous as well. Many economists have found empirical evidence for this assumption. For instance, Warner and Pleeter (2001) found in their study that individual discount rates vary between 0% and 70%. Gollier and Zeckhauser (2003) propose a way of aggregating time preferences in a group that leads to declining discount rates. One thing to note is that the focus of the above two papers is the pure rate of time preference PRTP.

The main conclusion of Gollier and Zeckhauser's (2003) study is that the social planner should not use a constant discount rate if individuals have heterogeneous constant rates of impatience (i.e. ρ in (3.7) is different for different individuals, but each individual's ρ remains the same over time) and if the planner aims to maximize the collective welfare of the group in a Pareto-efficient way. A Pareto-efficient way is one in which nobody can be made better off without making anybody else worse off. More specifically, if individuals exhibit decreasing *absolute risk aversions* (an index which is similar to μ in (3.7)), then the collective PRTP of the group is declining over time.

According to Gollier and Zeckhauser (2003), the Pareto-efficient way to allocate welfare is to let the most impatient members (those with the largest PRTP's in the group) have a larger share of the period's available wealth early in life and that share decrease with time and vice versa for patient members. Such a way is premised upon the fact that each period's 'cake' remains the same in size. So each individual's share of the 'cake' should sum up to the same during their lifetime. Therefore, those who gain a larger portion early on should have a share that's decreasing over time.

More formally, if $u(.)$ denotes the utility function of an agent, his tolerance to consumption fluctuations is measured by:

$$T = -u'(.)/u''(.) \tag{3.13}$$

where u' and u'' are the first and second derivative of *u(.)* in consumption, *T* is the inverse of absolute risk aversion *A*, i.e.:

$$T = \frac{1}{A} \tag{3.14}$$

It's shown that the collective rate of impatience equals a *weighted* mean of individual rates of impatience. The weights are proportional to the individual tolerances to consumption fluctuation, i.e. *T* in (3.13). The formal generalization of this proposition is:

$$\delta_{\upsilon}(z,t) = \frac{E\left[\delta(c,t,\theta)T(c,t,\theta)\right]}{E\left[T(c,t,\theta)\right]} \tag{3.15}$$

Where δ_{υ} is the collective PRTP for the group at time *t* with average per capita consumption at *z*. *E(.)* is the expectation operator.

If consumption is allocated in the Pareto-efficient way described above, patient individual's portion of the wealth increases with time whereas impatient ones' decreases. Therefore, when *T* increases with wealth, patient people (with small δ in (3.15)) will see their weights increasing in the collective rate, resulting in a declining social discount rate. The assumption that *T* increases with wealth is realistic on the same rationale as mentioned in 3.5.2 that people's aversion to risk decreases with wealth.

3.5.5 Problems with DDR

DDR provide a solution to the problem of long-term discounting but also have problems. The most notable of them is 'time inconsistency'. 'Time inconsistency' refers to the situation where plans made at one point in time are contradicted by later behaviour. This is a problem that has been observed long ago (e.g Ramsey, 1928) but is often credited to Strotz (1956) (OXERA, 2002). Although it's not uncommon for individuals to 'change their minds' about their earlier decisions, it poses a problem to social decision-makers.

A simple example will suffice. If ***A*** prefers one pound today to two pounds tomorrow, but prefers two pounds on the 51^{st} day to one pound on the 50^{th} day and designs a consumption plan accordingly, then when the 50^{th} day comes, ***A*** might decide to consume the one pound on that day instead of waiting until the 51^{st} day. Likewise, if one government decides to use high discount rates for the near future but lower ones for the far future, the immediate large spending will be easily justified. However, when later governments review the policy, they may decide that this earlier policy is not optimal and decide to increase the discount rate again, which will lead to higher consumption than planned. Therefore, if used 'naively', DDR's, especially hyperbolic discounting (because of its problem mentioned in 3.5.1 and Appendix 1), may lead to collapse of natural resources, as is shown by Hepburn (2003). The main reason for time inconsistency is that governments cannot commit their future counterparts- reviewing policies is almost certain and legitimate.

Heal (1998) proves that almost all types of declining discount rates result in time inconsistency. But Gollier and Zeckhauser (2003) suggest that their approach doesn't imply a similar problem. Each generation's decision-maker makes investment decisions only according to the current optimal allocation of wealth. Each individual still discount the future using a constant rate. Declining discount rates are only the 'natural' result of this process. However, this is conditional on the allocation of wealth according to the same Pareto-efficient way.

3.6 Previous applications of DDR

Applications of DDR in real policy appraisals have been limited, unlike theoretical researches. The biggest attempt has been made by the UK Treasury in its Green Book 2003, where the recommended social discount rates are declining with time (Table 3.2).

Table 3.2 The declining long term discount rate in the UK Green Book 2003

Period of years	0-30	31-75	76-125	126-200	201-300	301+
Discount rate	3.5%	3.0%	2.5%	2.0%	1.5%	1.0%

Source: UK HM Treasury (2003)

The recommendations in the Green Book are mainly based on studies that apply the uncertainty theory, such as that of Newell and Pizer (2003). In their study, two centuries of US interest rate data are used to quantify the effect of uncertainty. Under the random walk model, the certainty equivalent discount rate falls continuously from 4% to 2% after 100 years, 1% after 200 years and 0.5% after 300 years. Applying this scheme to an integrated assessment model, the RICE model created by Nordhaus et al., it is found the SCC is almost doubled.

Besides this study, there are other rough sensitivity studies on the effect of DDR on SCC. Some of them are listed in Table 3.3 below.

While these studies do give a concrete number to indicate the effect of DDRs, they apply different integrated assessment models and use different time horizons. Hence it's not convenient to compare them. Furthermore, the declining schemes they test are rather limited and the parameterizations of the schemes are rather simple.

Therefore, this thesis will carry out the sensitivity study in a more sophisticated way.

Using only one integrated assessment model, it is thus more convenient to compare the effects of different declining schemes.

Table 3.3 Some previous estimates on the effects of DDRs on SCC

Source	*Declining Scheme*	*Time Horizon*	*SCC estimate ($/tC)*
Nordhaus & Boyer (2000)	P_0= 3%, declining over time	2300	5.9
Newell & Pizer (2003)	S_0= 2%, random walk model	2400	33.8
	S_0= 4%, random walk model	2400	10.4
	S_0= 7%, random walk model	2400	2.7
	S_0=2%, mean-reverting model	2400	23.3
	S_0=4%, mean-reverting model	2400	6.5
	S_0=7%, mean-reverting model	2400	1.8
Tol and Heinzow (2003)	P_0= 3%, Weitzman discounting, falls to 1% after 200, 100, 50, 25 years	Not stated	0.8, 1.2, 1.7 and 2.1 respectively

Note: P_0 and S_0 refer to the initial pure rate of time preference and social rate of time preference respectively

Source: Adapted from Tol (2004)

3.7 Chapter summary

This chapter deals with issues in social discounting. Different candidates for the social discount rates and the difference between them are discussed. Four rationales for DDR are introduced to solve to problem of conventional discounting. The significance of the study in this dissertation is then discussed in the context of previous applications of DDR.

4. Updating FUND with new discounting schemes

4.1 Overview of the FUND model

4.1.1 General facts about FUND

The Climate Framework for Uncertainty, Negotiation and Distribution (FUND) is an integrated assessment model established in the late 1990s to estimate the global impacts of carbon dioxide emission. Since then, the model has undergone several updates and the current version is FUND 2.8.

FUND 2.8 divides the world into 16 regions, namely: Australia and New Zealand, Canada, Central America, China, Central and Eastern Europe, Former Soviet Union, Japan and South Korea, Latin America, Middle Africa, Middle East, South Asia, South-east Asia, Sub-Saharan Africa, Small Island States, USA and Western Europe.

For each of these 16 regions, FUND consists of a set of exogenous scenarios and endogenous perturbations. The exogenous scenarios concern economic growth, population growth, urban population, autonomous energy efficiency improvements, decarbonisation of energy use, nitrous oxide emissions and methane emissions. Although these scenarios are 'exogenous', they are nevertheless perturbed by climate change impacts, such as those resulting from changes in heat and cold stress, malaria. This implies that there is a small feedback within the model.

The endogenous parts include carbon dioxide emissions, the atmospheric concentrations of CO_2, methane and nitrous oxide, the global mean temperature, and the impact of climate change on coastal zones (sea level rise), agriculture, extreme weather (such as tropical cyclones), natural ecosystems and malaria. The emissions and concentrations of greenhouse gases are based on climate models, which form the scientific basis of the model. An important assumption in the model is that a doubling of the concentration of carbon dioxide equivalents will cause a 2.5℃ rise in global mean temperature.

The impacts are functions of both the climate change science and socio-economic variables and are derived from the literature (Tol, 2002a). The selection is based on a criterion that only those studies from General Circulation Models (GCMs) are used. Some impacts, such as agriculture and respiratory diseases can be positive or negative, depending on whether the climate is moving to or away from optimum. Others don't change sign and are modeled simply as power functions. The socio-economic variables have two units of measurement; people and money (Tol, 2003a). Population levels can change because of death or migration. All impacts are monetized.

The time span of the model is 1950-2300, in time steps of a year. The reason for extending the period back to 1950 is to initialize the model with past data. Also, since some adaptations (such as reduced vulnerability because of economic growth and technology progress) are included in FUND, some impacts depend on the impact of the year before. For a more detailed discussion on adaptations, see Tol (2003a, b). The period from 1950-1990 are based on historical observation while that from 1990-2100 is based on the FUND scenario, which lies somewhere in between the IS92a and IS92f (Tol, 2003a). Periods after 2100 are extrapolated from the trends in 2050-2100, which is a gradual shift to a steady state of population and economic growth. Such extrapolations are so far into the future that Tol himself recognizes that results from them are no to be relied upon (personal communication). However, such models are, by necessity, forward-looking, and the most one can ask for is to ensure that they continue to adapt with developments in climate science.

4.1.2 Discounting in FUND

The social discount rate used in FUND is the Social Rate of Time Preference (SRTP), constructed using the Ramsey Equation (equation (3.7)). In FUND, PRTP is assumed to be constant. Three discounting schemes using different PRTP's are applied, with values of 0%, 1% and 3%. μ is also constant and homogeneous for all regions, taking a value of 1.0. The only thing that varies with time is the growth rates of per capita GDP, g. Hence, a separate SRTP is calculated for each region in each year, based on the model prediction of GDP growth and population level. From this perspective, FUND already applies a time-varying discounting scheme, although not necessarily declining discount rates.

FUND also includes a simple declining discounting scheme. It applies Weitzman's discounting on the **pure rate of time preference (PRTP)**. The specification of the function is:

$$\rho(t) = \frac{0.375}{12.5 + t} \tag{4.1}$$

which implies that the corresponding mean and standard deviation in (3.12) are: μ=3% and σ= 4.89%.

4.1.3 Strengths and limitations of FUND for this research

The results of the sensitivity studies on declining discount rates depend on the choice of models. The choice of FUND as the tool of research offers several advantages.

First, FUND has been updated many times to reflect the most up to date knowledge in the literature. Several versions of FUND have been published and peer-reviewed, such as FUND 1.6 and FUND 2.0 (Tol, 2002a,b). Versions after 2.0 also take into account adaptive behaviour, which is regarded as a major improvement, although it has the danger of being 'too optimistic' (Clarkson and Deyes, 2002). Therefore, FUND offers a relatively sound knowledge base for comprehensive studies.

Second, FUND produces results that are region- and year- specific and it compiles the social discount rates from several parameters (i.e. ρ, g and μ) that are exogenous to the model. Such a design makes changing these parameters easy and allows separate values of each region to be applied. Specifically, the FUND 2.8 model divides the world into 16 regions, as compared with the 9 regions in previous versions. This is an appealing feature to a complicated sensitivity study on discount rates.

However, like other integrated assessment models, there are limitations with FUND. One can argue that uncertainties are so large that estimating the SCC is simply not possible, or at least not worthwhile. Putting this argument aside, there are other aspects of FUND that limit the full impact of different discounting schemes. One of these aspects is the treatment of catastrophes. Catastrophic events such as the shutdown of the thermohaline circulation or the collapse of the West-Antarctic Ice Sheet are not fully modeled. Admittedly, this is as much due to the limitation of currently available scientific research as it is to the FUND. For example, impact studies on the melt-down of the West-Antarctic Ice Sheet rarely go beyond sea level rise of more than 1 metre, when changes become non-linear. Since such impacts are not likely to happen in the near future, a declining discount rate will make them have larger influence on the present values of damages than with a constant discount rate. Therefore, the lack of 'catastrophes' paints an inaccurately optimistic picture of climate change which, relatively speaking, is even more inaccurate if declining discount rates are used.

The second limitation is that the scenarios in the model after 2100 are extrapolated and are highly speculative. However, for DDR to have effects, the time horizon should be much longer than 2100, when predictions are not dependable. Thus the creditability of such DDR studies is reduced. One way out of this dilemma is to compare the results of the same scheme from different models: if results of similar magnitude are obtained from different models, it at least gives confidence on the validity of the scheme being tested. Due to the availability of time and of different models, this method is not used here. Another possible way is **to compare the results before and after a declining discounting scheme is applied, with the same initial discount rates. This is the method used here.**

4.2 Implementing DDR from uncertainties

Since this dissertation aims to carry out a sophisticated sensitivity study, as many sensible schemes as possible should be tested. But some schemes are difficult to apply in practice. For example, hyperbolic discounting matches people's behaviour quite well, but the initial discount rates as high as 30%-40% are unrealistic for social policy making and hence this scheme is not used. DDR based on intergenerational equity such as those proposed by Chichilnisky (1997) and Li and Löfgren (2000) also have technical difficulties. The problem of the Chichilnisky formula is when the 'switching date' is (Dasgupta, 2001) and also choosing a weight for the near and far future. The difficulty with applying the Li and Löfgren (2000) approach is choosing discount rates for the conservationist and the utilitarian and the weight to place upon them. Proxies of these parameters are either absent or very difficult to find in reality. Further illustration of these problems can be found in Appendix 1, 3 and 4. Given these difficulties, the dissertation only chooses those schemes that give directions for practical application. Two categories of schemes are chosen: DDR based on uncertainties and DDR based on heterogeneous preferences. This section deals with the first category and the next section with the second one.

4.2.1 UK Green Book discounting

The 'Green Book Appraisal and Evaluation in Central Government' published by the UK HM Treasury in 2003 is an update of its previous versions. The aim of the document is to provide guidance on public project appraisals 'before significant funds are committed' (HM Treasury, 2003). Two significant features concerning discounting are included in this version. First of all, the new edition "unbundles" the discount rate, introducing a rate of 3.5% in real terms, based on social time preference, while taking account of the other factors which were in practice often implicitly bundled up in the old 6% real figure.

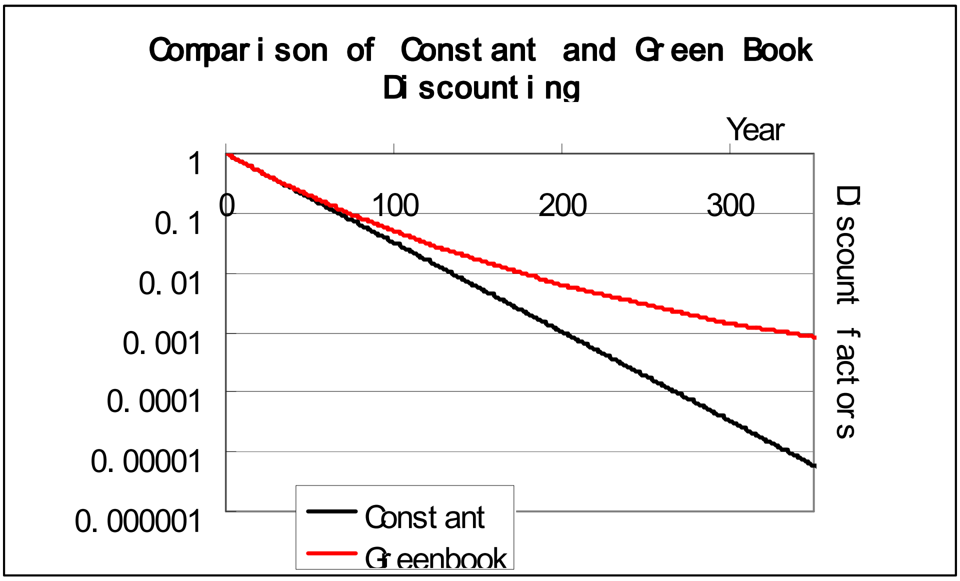
Second, for the first time, a declining discounting scheme (Table 4.1) is introduced to evaluate projects with very long-term impacts, i.e. over thirty years. The scheme of the Green Book discounting can be shown by a comparison with a traditional constant discounting scheme, as in Figure 3.1 below. Note that the y-axis is in logarithmic scale.

Table 4.1 The declining long term discount rate in the UK Green Book 2003

Period of years	0-30	31-75	76-125	126-200	201-300	301+
Discount rate	3.5%	3.0%	2.5%	2.0%	1.5%	1.0%

Source: UK HM Treasury (2003)

Figure 4.1 Comparison of discount factors from constant and Green Book discounting



The method to update the FUND with the Green Book discounting scheme is to use the discount rates recommended for each year after 2000, and for the world as a whole. In other words, one social discount rate is used for the whole world in each year, rather than using region-specific discount rates in the previous FUND model. One reason for this way of applying the scheme is that the discount rates in the scheme are SRTP rather than PRTP, and hence it's better not to disintegrate them.

One problem for applying the scheme in this way is that a UK-based scheme is applied on other regions of the world. However, some rationales can justify it. First, discount rates used in developed countries are not greatly different from each other and are often lower than those in developing countries. This is due to the fact that many developing countries have a higher economic growth rate (i.e. the *g* in the Ramsey Equation is larger for developing countries). Yet impacts of future climate changes are likely to strike developing countries harder because of their vulnerabilities. So using lower discount rates for developing countries is in their interest and also makes sense to developed countries.

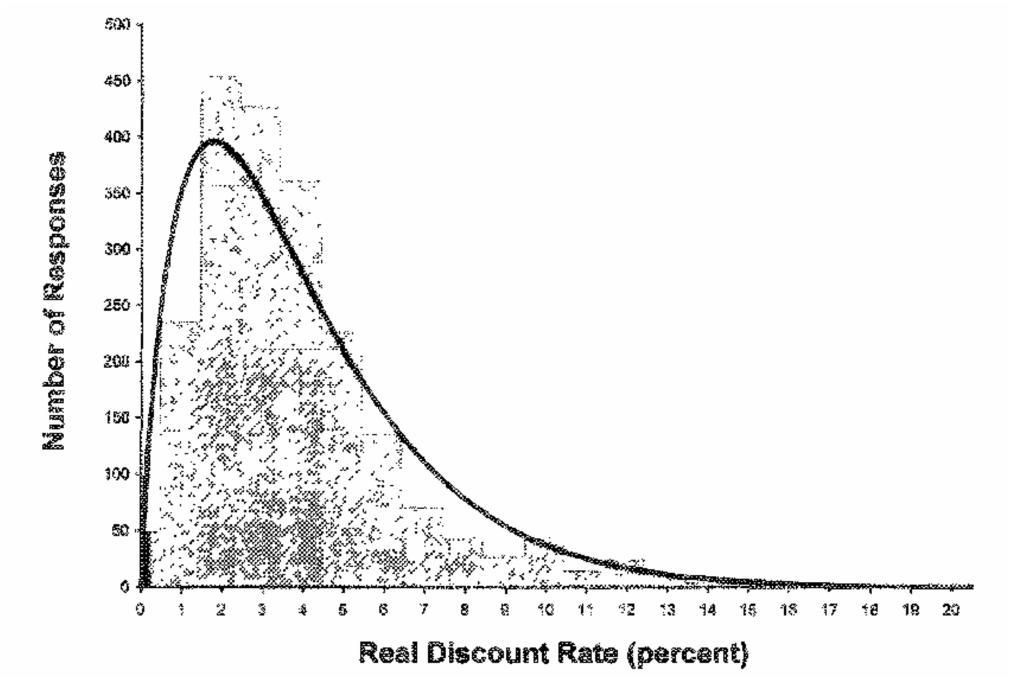
Second, the scenarios for economic growth and population in FUND start to converge after 2100. This implies that after 2100, using a global discount rate won't produce many deviations from using region-specific ones.

Therefore, the implementation of the Green Book discounting on all regions of the world in FUND 2.8 is to a large extent justified and worthwhile.

4.2.2 Weitzman discounting

In section 3.5.2, the certainty equivalent discount rate is expressed as: $R(t) = \frac{\mu}{1 + t\sigma^2 / \mu}$, where μ and σ are the mean and standard deviation of the gamma distribution respectively. In order to estimate the current 'uncertainty' about the discount rate to be used on the far distant future, Weitzman carried out an email survey on over 2000 economists around the world. The 2160 responses he got display a gamma distribution, as in Figure 4.2 below. Note that the discount rates are rounded to the nearest whole percentages.

Figure 4.2 Illustration of gamma distribution of discount rates



Source: Weitzman (2001)

Weitzman also did a broken-out subsample of 50 'blue-ribbon' distinguished economists. The mean and the standard deviation calculated from these two surveys match very well and hence Weitzman proposes two rounded-off average values as parameters for the gamma distribution: $\mu = 4\%$ per annum and $\sigma = 3\%$ per annum.

If Weitzman's 'uncertainty' about discount rates can also be interpreted as different 'opinions' about the rates to use, then his survey result is most valuable and of high-quality (criticism and doubts exist, though, e.g. Dasgupta, 2001). Therefore, his results seem convincing and worth trying out.

Although FUND has included a simple DDR scheme based on Weitzman's proposition, the implementation proposed here is not just repetitive because there are two improvements in methodology. First, DDR will be applied on SRTP instead of PRTP as was done previously. Although Weitzman didn't specify what rate he was talking about, it can be deduced from his survey questions that he probably meant SRTP. The operative part of his question in the survey was: 'Taking all relevant considerations into account, what real interest rate do you think should be used to discount over time the (expected) benefits and (expected) costs of projects being proposed to mitigate the possible effects of global climate change?' (Weitzman, 2001) The 'interest rate' is used directly on the benefits and costs of climate change, so it should be the SRTP. Second, the parameters tested here are more soundly-based because Tol himself acknowledged that his specification (μ = 3% and σ = 4.89%) is somewhat *ad hoc*.

Therefore, **the first way to implement the Weitzman discounting in FUND is to apply a discount rate for the world in each year that declines according to the following formula:**

$$R(t) = \frac{0.04}{1 + t0.03^2 / 0.04} = \frac{4}{100 + 2.25t} \tag{4.2}$$

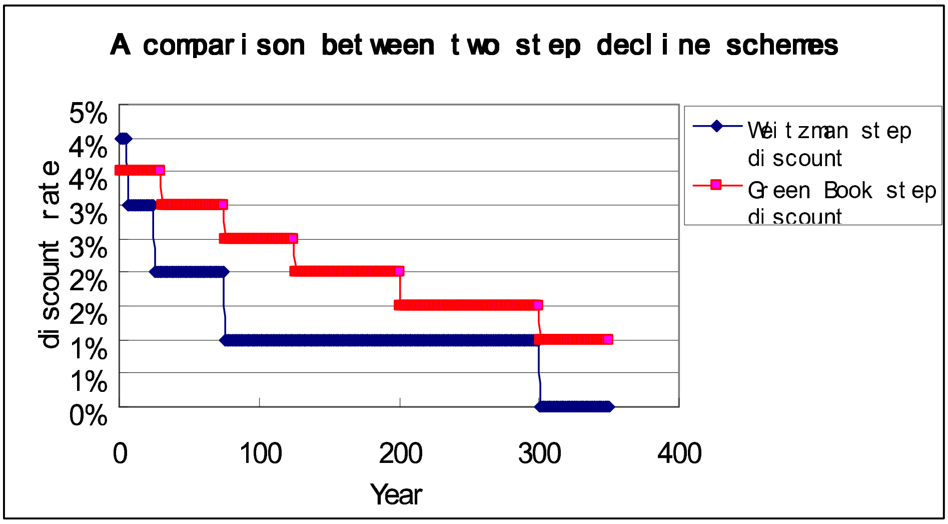
Furthermore, since Weitzman also suggests a step-declining schedule to approximate the gamma discounting function, it is also possible to test this schedule in the FUND model to see how closely these two schedules match. Because the step-declining schedule has a higher practicability, a close match would further strengthen the applicability of Weitzman discounting. The schedule suggested is shown in Table 4.2 and a comparison between the Weitzman step decline and the Green Book discounting is shown in Figure 4.3. It can be seen that the discount rates in the Weitzman scheme are overall lower than the Green Book discounting so it can be expected that Weitzman's scheme would give a higher SCC number.

Table 4.2 The Weitzman step decline schedule to approximate gamma discounting

Period of years	1-5	6-25	26-75	76-300	301+
Discount rate	4%	3%	2%	1%	0%

Source: Weitzman (2001)

Figure 4.3 A comparison between Weitzman and Green Book step decline



Source: After Weitzman (2001) and HM Treasury (2003)

4.2.3 Gollier's discounting

In Weitzman's discounting, the equilibrium SRTP as a whole is taken as uncertain. Gollier (2002a,b) takes a different approach by breaking down the SRTP and treating the consumption growth (g) as uncertain. Gollier's approach involves much more complicated specifications about individual utility functions and risk preferences. That's why more restrictions are needed to get declining discount rates. Since it is unrealistic to test Gollier's assumptions, this paper implements DDR from growth uncertainties by using the central idea of Gollier's paper and adapting it in a more practical context.

The basic idea is that if there are more than one possible growth rates in the future period, the *discount factor* for that period should be a weighted sum of all the possible *discount factors* and the weight is the probability of each growth rate. This is a similar idea as the one in Weitzman (1998). The only difference is the thing we're uncertain about- the discount rate or the growth rate.

A simple way to implement this idea is to change the certain growth rates of per capita income in FUND into uncertain. The first way of changing this is to set an upper and lower boundary for the per capita income growth rates, using the original 'certain' growth rates as the mean estimates. This scheme can be shown as below:

(i) if the original prediction of growth rates is $g_0 = a\%$, and the uncertainty boundary is b%, then the new possible growth rates for that year are: $g_l = (a\text{-}b)\%$ and $g_u = (a+b)\%$ respectively.

(ii) These two new growth rates can be used to calculate new discount rates (SRTP) for that year, according to the Ramsey equation:

$$\begin{aligned} SRTP_l &= PRTP + \mu \cdot g_l \\ SRTP_u &= PRTP + \mu \cdot g_u \end{aligned} \tag{4.3}$$

Where $SRTP_l$ and $SRTP_u$ are the lower and upper SRTP calculated using the two new growth rates above.

(iii) A 'certainty equivalent SRTP' is calculated by weighting the two SRTPs above, assuming an equal probability for each of them:

$$SRTP_e = \frac{1}{2}(SRTP_l + SRTP_u) \tag{4.4}$$

Where $SRTP_e$ is the weighted SRTP.

(iv) the $SRTP_e$ is then used as the new discount rates to be used for that year in that region.

The boundaries ('*b*') have been set as 1% for all years and all regions. This specification is somewhat *ad hoc* and therefore it's more illustrative than conclusive. However, it is a useful first step.

The second scheme is to assume that the probability distribution of uncertain growth rates is that of a normal distribution, with the original growth rate in FUND as the mean value, μ, and suppose the standard deviation of the distribution is σ. Five discrete points are used to approximate the normal distribution. These five points are: μ-2.5σ, μ-1.5σ, μ+1.5σ, μ+2.5σ and μ. Their probabilities are as follows:

$P(\mu\text{-}2.5\sigma) = P(\mu+2.5\sigma) = 1\text{-}\Phi(3)+ \Phi(3)\text{-} \Phi(2) = 1\text{-}\Phi(2) = 0.0227$

$P(\mu\text{-}1.5\sigma) = P(\mu+1.5\sigma) = \Phi(2)\text{-} \Phi(1) = 0.9773\text{-}0.8413= 0.136$

$P(\mu) =2(\Phi(1)\text{-} \Phi(0)) = 2(0.8413\text{-}0.500) = 0.6826$

Where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{1}{2}u^2\right)du$, which is the standard normal distribution function.

The standard deviation of the distribution is set to be 0.5%, which roughly has the same upper and lower boundary as the above two-point scheme. The calculation of a certainty equivalent discount factor is similar to the previous one.

(i) Five SRTP's are calculated, which are based on five possible growth rates, where g is the original growth rate in FUND.

$$SRTP_1 = PRTP + \mu(g - 2.5\sigma)$$
$$SRTP_2 = PRTP + \mu(g - 1.5\sigma)$$
$$SRTP_3 = PRTP + \mu g$$
$$SRTP_4 = PRTP + \mu(g + 1.5\sigma)$$
$$SRTP_5 = PRTP + \mu(g + 2.5\sigma)$$

(ii) A 'certainty equivalent SRTP' is calculated by weighting these five SRTPs, using the probabilities above:

$$SRTP_e = 0.0227(SRTP_1 + SRTP_5) + 0.136(SRTP_2 + SRTP_4) + 0.6826 \cdot SRTP_3 \quad (4.5)$$

(iv) the $SRTP_e$ is then used as the new discount rates to be used for that year in that region.

4.2 Implementing DDR from heterogeneous time preferences

To implement Gollier and Zeckhauser (2003)'s proposition on heterogeneous time preferences, it is necessary to have the following information: (1) the utility functions of individual agents in the group; (2) the way individual agents discount future; (3) the distribution of the time preferences (PRTP) within the group; and (4) the Pareto-efficient social welfare function to allocate wealth within the group. This section first deals with the assumption with regards to each of these four categories and then come up with a way to implement the DDR in FUND.

4.3.1 Utility functions of individual agents

The concept of *'utility'* is used in economics to represent the advantage or fulfilment a person receives from consuming a good or service. It explains how individuals and economies aim to gain optimal satisfaction in dealing with scarcity. Utility is an abstract concept rather than a concrete, observable quantity. The units to which we assign an 'amount' of utility, therefore, are arbitrary, representing a relative value.

The *utility function* is there to describe how our sense of 'fulfilment' or 'happiness' are determined by various arguments. It follows from the above definition of utility that consumption is one of the most important, although not the only, arguments of the utility function. Because other parameters describing inter-temporal behaviours, such as 'risk aversion', are dependent on the utility function, defining a utility function is crucial to determine how these parameters change with consumption.

Strictly speaking, it is very difficult, if not impossible, to define a utility function for each person (or 'agent') in the group. Nor is it realistic to assume that everybody uses the same utility function. But to facilitate research, it is necessary to make some simplified assumptions about people's preferences and utility functions. One of the most frequently used utility functions in economics is a group of 'iso-elastic' utility functions. These functions are in the form of (4.6):

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{4.6}$$

where u is utility, c is consumption and γ is the 'relative risk aversion'.

The more formal definition of 'relative risk aversion' (R) is:

$$R = -c \cdot \frac{u''(c)}{u'(c)} \tag{4.7}$$

where *c* is consumption and *u'(c)* and *u"(c)* are the first and second derivative of the utility function in consumption. For iso-elastic utility functions, it can be checked that $R = \gamma$. It can also be derived that when γ tends to 1, the utility function will become:

$$u(c) = \ln c \tag{4.8}$$

Under this assumption of utility function, the individual tolerance to consumption fluctuations over time can be shown as:

$$T(c) = -u'(c)/u''(c) = -\frac{c^{-\gamma}}{-\gamma \cdot c^{-\gamma-1}} = c/\gamma \tag{4.9}$$

and $$T'(c) = 1/\gamma \tag{4.10}$$

where *T'(c)* is the first derivative of tolerance.

It is a standard assumption in economics that γ is positive and hence *T'(c)* in (4.10) is positive. Since T is the inverse of risk aversion A (see equation (3.14)), (4.10) further implies that iso-elastic utility functions have the property of decreasing absolute risk aversion, which is necessary in getting a declining collective PRTP in a group with heterogeneous PRTPs.

Because iso-elastic utility functions capture some salient features of human needs and are relatively simple to work with, they are often used in economic modelling. Following this tradition, it is assumed here that individuals in the group have the same iso-elastic utility functions[4] in the form of (4.8).

4.3.2 The way individuals discount future

As discussed in 3.5.1, there is evidence that individuals discount future utilities hyperbolically. Yet hyperbolic discounting encounters the typical time consistency problem. Economics tends to assume that people are rational and thus the individual decision-making process should be self-consistent. Therefore, it's reasonable to assume here that individuals discount future utilities using a constant PRTP. This is also interpreted as individuals having homogeneous beliefs about the future (Wilson, 1968).

4.3.3 The distribution of PRTP

To estimate the collective PRTP in equation (3.15) is to calculate the weighted mean of heterogeneous PRTPs in the group. So it is crucial to know what PRTP each individual in the group uses, i.e. the distribution of PRTP in the group.

---

[4] Despite this, each person still use a different PRTP to discount their own utility function.

However, as discussed in OXERA (2002), PRTP is ‘the least amenable to empirical analysis’ in the Ramsey equation and estimating the value of PRTP is not easy, let alone the distribution of PRTP. Based on Gollier and Zeckhauser (2003)’s example, it is assumed here that the PRTPs follow a negative exponential distribution:

$$\delta \sim f(\delta) = e^{-\delta/\mu} / \mu, \delta \in [0, +\infty] \tag{4.11}$$

where δ is individual agents’ PRTPs, μ is the mean of the negative exponential distribution, i.e. $E\delta = \mu$. This assumption is somewhat *ad hoc* but is a good starting point to work with. And it also corresponds to the literature’s general assumption that PRTPs are non-negative.

The next step is to estimate the mean of the distribution. Difficult to estimate though, some attempts have been made to deduce the national PRTP from empirical data. For example, Scott (1977) used the UK Consols (safe long-term government bond) yield (as SRTP) between 1855 and 1914 and the growth of net national income per head (as g in Ramsey equation) to estimate PRTP. Long-term government bond yields are seen as good proxies of SRTP because they have the longest time span, although still not exceeding 30 years. Following this pathway, the ideal way to estimate the PRTP distribution is to search for similar researches as Scott‘s (1977) and construct a database for as many countries as possible. Unfortunately, such data are limited to a few developed countries so the data set is not large enough to depict a proper distribution.

On the belief that it is better to have a large sample free of availability bias than to have a small one focused on a few countries, this paper proposes a ‘second best’ approach to estimate the PRTP in the world. The idea is to use the government central bank discount rates as proxies of SRTP. Data on this parameter are readily available from the International Monetary Fund (IMF) database 'International Financial Statistics (IFS) (2003)', covering the period from 1948 to 2002 and 138 countries in the world (some countries for a shorter period of time). Central bank discount rate is defined as 'the rate at which the central banks lend or discount eligible paper for deposit money banks, typically shown on an end-of-period basis'. It reflects the government discount rate for shorter period than the government bond, but still reflects the social preference. It's a 'second best' proxy in terms of reflecting long-term orientation but a better proxy in terms of data availability because the IMF database on government bond yield only covers 48 countries in the world. A compromise between sample size and accuracy would favor the adoption of the central bank discount rates.

Since the central bank discount rates are used to discount 'paper' money, inflation would affect the level of the rates. Taking this into account, data on inflation, i.e. the rise of consumer prices per annum, are used to deflate the discount rates before further calculation. Data on consumer prices are also available from the IMF IFS database.

For proxies of per capita income growth, GDP per capita growth rates are used. This is consistent with the general approach in economics and the assumption in the FUND model because it is also used in the FUND calculation of SRTP. Data are obtained from the World Bank World Development Indicators database and cover the time period from 1961 to 2002. Because the annual growth rates in the World Bank database are growth rates of real terms, deducting inflations, so no deflation is required for this parameter.

The overlap of the IMF and World Bank database determines that the longest time period available for estimation is the 42 years from 1961 to 2002, long enough to give a reliable mean value of PRTP, although for quite a few countries, the time series is much shorter.

Assuming μ =1 (income elasticity of marginal utility in the Ramsey equation), the PRTP can then be estimated as:

$$\rho = s(1-c) - g \qquad (4.12)$$

Where ρ is the PRTP, s is the central bank discount rate per annum, c is the inflation of consumer prices per annum and g is the GDP per capita growth rate per annum.

An unweighted long-term mean for each country is then calculated. And in line with the division of countries in FUND, the final data set contains the following 113 countries listed by regions. Note that some countries are deemed as ‘outliers’ in the original data set and are omitted because they have exceptionally high or low PRTPs, larger than 20% or smaller than -10%. It is regarded as not practical to apply such extreme PRTPs in public decision making. The possible reason for such extreme numbers is those countries have undergone drastic changes in their economies, such as some Eastern European and former Soviet Union countries. Hence during the time period considered, their data on the above parameters fluctuated too much and didn’t reflect the ‘true’ PRTP of those countries.

The mean PRTP for each country is rounded to the nearest whole percentages. The distribution of the 113 PRTPs is shown in Table 4.4 and Figure 4.4 is the histogram of these data. Further details of the PRTPs of each country can be found in Appendix 5.

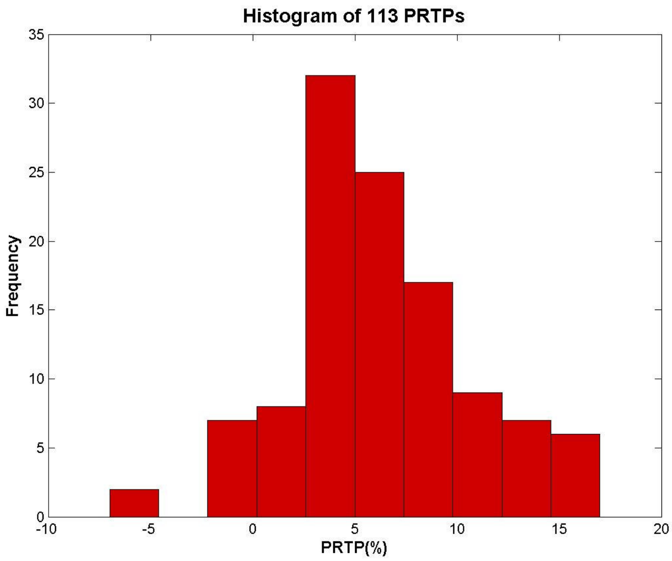
Table 4.3 List of the 113 countries in the final PRTP data base

Region	*Number of countries*	*Name of countries*
ANZ (Australia and New Zealand)	2	Australia, New Zealand
CAM (Central America)	2	Belize, Costa Rica
CAN (Canada)	1	Canada
CHI (China)	1	China
EEU (Eastern Europe)	6	Albania, Croatia, Czech Republic, Hungary, Poland, Slovenia
FSU (Former Soviet Union)	2	Latvia, Russia
JPK (Japan and South Korea)	2	Japan, South Korea
LAM (Latin America)	9	Bolivia, Chile, Colombia, Dominica, Ecuador, Guyana, Paraguay, Peru, Venezuela
MAF (Middle Africa)	5	Algeria, Egypt, Libya, Morocco, Tunisia
MDE (Middle East)	5	Israel, Jordan, Kuwait, Lebanon, Syrian, Yemen
SAS (South Asia)	5	Bangladesh, India, Nepal, Pakistan, Sri Lanka
SEA (South East Asia)	6	Indonesia, Lao, Malaysia, Myanmar, Philippines, Thailand, Vietnam
SIS (Small Island States)	15	Antigua and Barbuda, Bahamas, Barbados, Cyprus, Fiji, Grenada, Malta, Netherlands Antilles, Papua New Guinea, St Kitts and Nevis, St Lucia, St Vincent and Grens, Vanuatu Trinidad and Tobago,
SSA (Sub-Saharan Africa)	34	Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central Africa, Chad, Congo, Republic of, Cote d'Ivoire, Equatorial Guinea, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Malawi, Mali, Mauritius, Mozambique, Namibia, Niger, Nigeria, Rwanda, Senegal, Seychelles, South Africa, Swaizland, Tanzania, Togo, Uganda, Zambia, Zimbabwe
USA	1	United States of America
WEU (Western Europe)	16	Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK

Table 4.4 Distribution of the113 PRTPs

PRTP (%)	-7	-5	-2	0	1	2	3	4	5	6	7
Frequency	1	1	2	5	2	6	6	13	13	12	13
PRTP (%)	8	9	10	11	12	13	14	15	16	17	
Frequency	9	8	5	1	3	4	3	2	2	2	

Figure 4.4 Histogram of the 113 PRTPs



If the negative PRTPs in the sample are not taken as 'outliers', an unbiased estimator, the *minimum variance unbiased estimator* (MVUE) is then used to estimate the parameters of the data. The MVUE has the minimum variance of all unbiased estimators of a parameter. **The mean of the sample are estimated as 6.4% per annum.**

Several points are worth mentioning from the results of the data set. First, in the data set, PRTPs in developing countries are generally higher than those in the developing countries. For example, the mean PRTP for the Western Europe and USA are 4.25% and 3.33% while that of the Sub-Saharan Africa is 7.52%. This might be due to the higher economic growth rates in developing countries. Another possible reason can be referred to Pearce and Ulph's (1999) account of the PRTP (see equation (3.8)). For some Sub-Sarahan countries, political instability might render a negative value of *L* in equation

(3.8), meaning that life chances get smaller through time. This would increase the overall PRTP in (3.8).

The second point is that the mean PRTP is much higher than what's suggested in the literature (e.g. the UK Green Book suggests 1.5%). The higher PRTPs in developing countries push up the mean. Further more, many researches on PRTPs were carried out in the developed world, which would drive a bigger wedge between the literature estimates and the estimates here.

Third, even the mean for the countries in the developed world is higher than in the literature. For instance, the mean for the UK is 6%, much higher than the recommended 1.5% in the Green Book. The possible explanation is the methodology difference. The central bank discount rate might not be as good as other proxies such as the government bond yields used in the literature.

Despite all these limitations, the mean value is not too high to use in practice. Therefore, the mean value of 6.4% is used in subsequent estimates.

If PRTPs are assumed to be non-negative, the 9 negative PRTPs in the results become a bit problematic. The treatment here is to set those to zero and the new mean value is estimated as 6.6%, a small difference from the previous one. Therefore, only 6.4% is used here.

4.3.4 The social welfare function

The social welfare function determines how individual agents' utilities are weighted in the group. Following Goller and Zeckhauser (2003), the assumed weighting function takes the form of:

$$\lambda(\theta) = \theta^{\eta} \tag{4.13}$$

Where heterogeneous agents are indexed by θ in a type set Θ and η is some scalar. One conception of fairness when all agents have the same utility function would set $\eta = 1$. This implies that the mean weight of individuals' utility over their (infinite) lifetime is the same for everyone (Gollier and Zeckhauser, 2003):

$$\int_0^{\infty} \lambda(\theta) e^{-\theta t} dt = 1, \forall \theta \in \Theta \tag{4.14}$$

4.3.5 *The collective PRTP for the world*

To sum up, the individual agents in a group, i.e. the 16 regions in the FUND model, are assumed to have the same iso-elastic utility function of: *u(c) = ln (c)*. And agents discount future utilities with constant but heterogeneous PRTPs. The distribution of PRTPs is assumed to be negative exponential: $\delta \sim f(\delta) = e^{-\delta/\mu} / \mu, \delta \in [0, +\infty]$. The weighting function of agents takes the form of $\lambda(\theta) = \theta^{\eta}$, where η is assumed to be 1.

Using equation (3.15), it can be derived that:

$$\delta_v(t) = \frac{\eta + \gamma}{t + \frac{\gamma}{\mu}} = \frac{1+1}{t + \frac{1}{\mu}} = \frac{2}{t + \frac{1}{\mu}} \tag{4.15}$$

Where δ_υ is the collective PRTP, *t* is time and μ is the mean of the distribution and will take the value of 6.4%. (4.15) is the scheme to be applied in the FUND model, on all regions each year. Each region will have the same PRTP, but still a different per capita growth rate of consumption. So the final SRTP for each region in each year is still different for each region.

4.4 Chapter summary

This chapter first discusses the general features of the FUND 2.8 model and its specific feature related to discounting. Then, five ways to implement DDR in the model are proposed: the Green Book step-declining scheme, Weitzman step-declining scheme, Gamma discounting, Gollier discounting from uncertain growth and DDR from heterogeneous time preferences (Gollier heterogeneous discounting). The implementation of the first three schemes is based on the direct application of the results in relevant literature while the implementation of remaining two is the novel proposition in this paper. These schemes can be summarized in the Table 4.5 on next page. The next section will discuss the results of the implementation of these schemes.

Table 4.5 Summary of the DDR schemes to update FUND

Scheme Name	*Way of implementation*
Green Book step-declining scheme	1. Use one discount rate (SRTP) for the world in each year after 2000 2. The SRTPs are based on and decline according to the recommended step decline scheme in the UK Green Book 2003
Weitzman step-discounting scheme	1. Use one discount rate (SRTP) for the world in each year after 2000 2. The SRTPs decline according to the suggested step decline scheme in Weitzman (2001)
Gamma discounting	1. Use one discount rate (SRTP) for the world in each year after 2000 2. The SRTPs decline according to the suggested gamma function in Weitzman (2001)
Gollier discounting from uncertain growth	Scheme 1: Calculate a 'certainty equivalent SRTP' using two possible growth rates, calculated by extending the original growth rates with an upper and lower boundary of 1%. Apply the new SRTP instead of the original one. Scheme 2: Calculate another 'certainty equivalent SRTP' using five possible growth rates in the five scenarios in FUND 2. Apply the new SRTP instead of the original one.
DDR from heterogeneous time preferences (Gollier heterogeneous discounting)	1. Agents are assumed to have homogeneous iso-elastic utility functions and discount the future using constant PRTPs 2. PRTPs in the world is assumed to be negative exponential. 3. The mean value of the distribution is estimated based on a survey on 113(110) countries. Central bank discount rates, consumer prices index and GDP per capita growth are used as proxies to calculate PRTP for each country 4. Assumption about the social welfare function is made. 5. A collective PRTP for the world that declines with time is then derived 6. The collective PRTP is then applied on all regions, with the GDP per capita growth of the each region to calculate the SRTP for each.

5. Results and Discussion

5.1 SCC from constant PRTPs

For comparison, the SCC from constant PRTPs in FUND 2.8 are listed in Table 5.1. The numbers in this chapter are expressed as US dollars per ton of carbon. The numbers in this chapter are mostly obtained without equity weighting, unless otherwise specified.

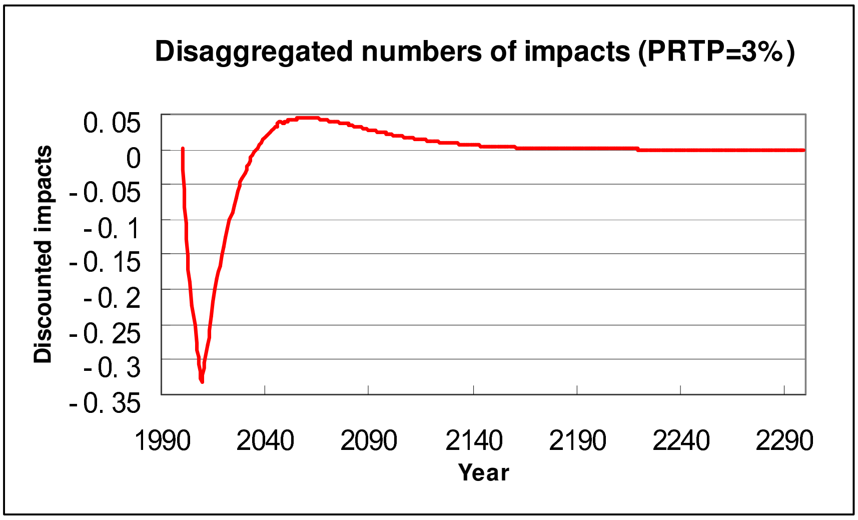
Table 5.1 SCC from constant PRTPs

PRTP (%)	*SCC ($/tC)*
0	58
1	11
3	-2.3

SCC from a constant PRTP of 3% is negative, which means that an extra ton of carbon emission today will lead to benefits. This is due to the damage profile in FUND- the near-term effects are dominated by benefits (e.g. increase of agricultural productivity) while damages are in the far-distance future. This can be shown by disaggregating the discounted impacts to each year (Figure 5.1). It shows that the impacts of increased CO_2 are mainly beneficial before 2040 but become damages after that. Therefore, under higher discount rates, the damages in the far distant future are counted less than the benefits in the near future.

The SCC numbers below are the results of different combinations of DDR schemes and PRTPs, which produce 10 sets of results altogether. Some of them are negative as well and they can be more or less explained by this damage profile in FUND 2.8.

Figure 5.1 Disaggregated numbers of impacts (discounted with PRTP=3%)



5.2 Results from step-declining schemes

As mentioned in section 4.1.3, the method to test the effects of DDR schemes in this paper is to compare the results before and after a DDR scheme is applied, with the same initial discount rates. Therefore, the results listed below include the results from the same starting discount rates which are constant. The 'percentage of SCC increased' refers to the increase in SCC after declining discount rates are applied, compared with the constant scheme. Table 5.2 shows the results of the two step-discounting schemes.

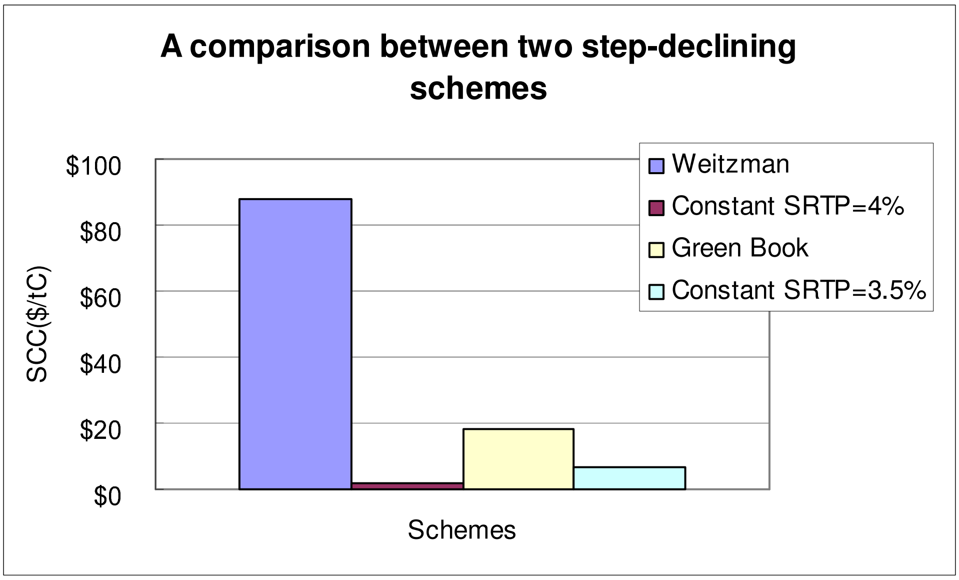
One of the major sources of theoretical basis for the Green Book step declining scheme is Newell and Pizer (2003). In their paper, a declining scheme starting from 4% is applied to Nordhaus' RICE model. It is found that the percentage of SCC increased is 82% under the random walk model, compared with the 174% here. The difference in model specification is one possible reason for this difference. As noted above, FUND 2.8 displays a big difference between benefits and costs of CO_2 before and after 2040 while in the RICE model, this difference is smaller. A DDR would therefore increase the weights of future costs more and have a significant effect on the final number. However, although the dollar value of the SCC is sensitive to the magnitude of the benefits profile in different models, the proportional increase due to DDR are generally significant, irrespective of the underlying model.

Table 5.2 SCC results from step-declining discounting schemes

Scheme		*SCC ($/tC)*
Green Book	SCC value ($/tC)	18
Step-declining	Constant SRTP = 3.5%	6.6
	Percentage of SCC increased	174%
Weitzman	SCC value ($/tC)	88
Step-declining	Constant SRTP = 4.0%	2.1
	Percentage of SCC increased	4100%

Figure 5.2 is a comparison of the results from the two step-declining schemes. Compared with the Green Book discounting scheme, Weitzman's scheme increases the SCC more drastically. This is also what's expected in 4.2.2 (see Figure 4.3 on P.41) because Weitzman's scheme declines faster than the green book scheme, although it starts from a slightly higher discount rate.

Figure 5.2 A comparison between the Green Book and Weitzman step-declining scheme



The increase in SCC here is probably conservative insofar as time span is concerned because the discount rates in both schemes fall to even lower levels after 300 years, which is out of the time span in FUND 2.8. Nevertheless, one can argue that predictions that far into the future are not dependable.

5.3 DDR from gamma discounting

The results of DDR from gamma discounting are listed in Table 5.3, together with the results from Weitzman step decline for comparison.

Table 5.3 SCC results from gamma discounting

Discounting Scheme	*SCC ($/tC)*
Gamma Discounting	88
Constant SRTP = 4.0%	2.1
Percentage of SCC increased	4100%
Weitzman Step Declining	88

The results show that the gamma discounting and the Weitzman step-declining scheme match very well. In other words, the step-declining scheme can be a good guide for policy and practical implementation if Weitzman's rationale for declining discount rates is to be adopted.

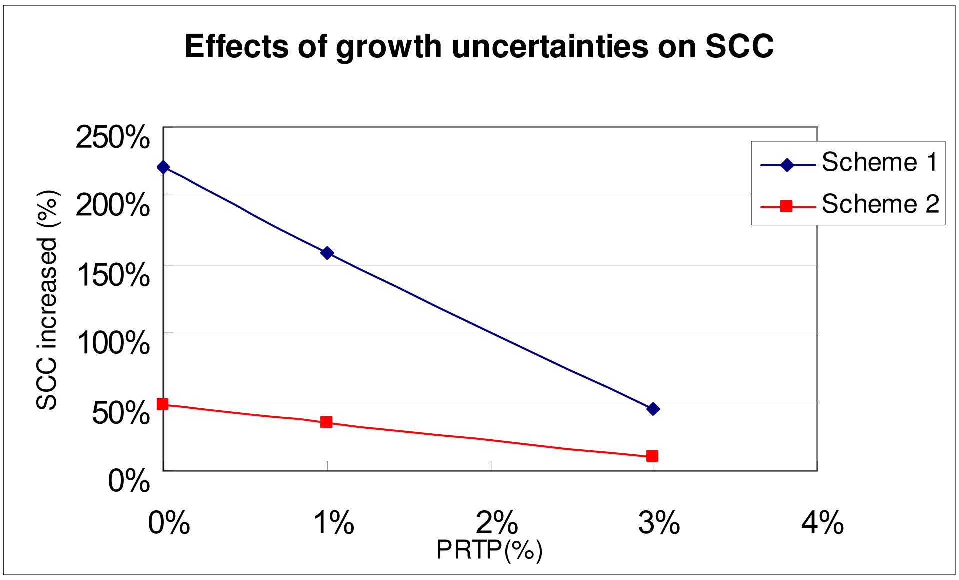
5.4 DDR from growth uncertainties

Table 5.4 lists results from the two schemes based on growth uncertainties, as described in 4.2.3 (P. 42-43).A phenomenon can be found in Table 5.5 and further illustrated in Figure 5.3: the higher the constant PRTP is, the less effect growth uncertainties have on the SCC numbers.

Table 5.4 SCC Results from DDR based on growth uncertainties

		SCC ($/tC)		
Constant PRTP		0%	1%	3%
Scheme 1	SCC value	185	29	-1.3
	Percentage increased	221%	159%	45.7%
Scheme 2	SCC value	85	15	-2.1
	Percentage increased	48%	35%	10%

Figure 5.3 Effects of the growth uncertainties on SCC



The explanation can be given by a simple mathematical illustration. Suppose the discount factor for year t and t-1 are DF_t and DF_{t-1} respectively. The constant PRTP is $a\%$, the growth rate of per capita income from year t-1 to t is $g\%$. Assuming a 1% uncertainty upper and lower boundary in the growth rate, DF_t in this experiment is calculated in the following way:

$$(DF_t)_u = \frac{DF_{t-1}}{1+a\%+g\%-1\%}$$

$$(DF_t)_l = \frac{DF_{t-1}}{1+a\%+g\%+1\%}$$

$$DF_t = \frac{1}{2}\left((DF_t)_u + (DF_t)_l\right) = DF_{t-1} \cdot \frac{\left(1+a\%+g\%\right)}{\left(1+a\%+g\%-1\%\right)\left(1+a\%+g\%+1\%\right)}$$

By comparison, if the discount factor is calculated in the 'old way', it is expressed as:

$$DF_t^{'} = \frac{DF_{t-1}}{1+a\%+g\%}$$

The increase in the discount factor can be expressed as the following ratio:

$$R = \frac{DF_t}{DF_t^{'}} - 1$$

Further assume that there are two PRTPs, $a_1\%, a_2\%(a_2 > a_1)$, the discount factors at time t based on these two PRTPs are represented as DF_1 and DF_2. And the increase in the discount factor is R_1 and R_2. It can be checked that:

$$\begin{aligned} R_1 - R_2 &= \frac{DF_1}{DF_1^{'}} - \frac{DF_2}{DF_2^{'}} \\ &= 1\% \cdot \frac{\left(a_2\%+1+g\%\right)^2 - \left(a_1\%+1+g\%\right)^2}{\left(a_1\%+1+g\%-1\%\right)\left(a_1\%+1+g\%+1\%\right)\left(a_2\%+1+g\%+1\%\right)\left(a_2\%+1+g\%-1\%\right)} > 0 \end{aligned}$$

In other words, the higher the PRTP is, the less the weights on future value are increased. This corresponds to the results in Table 5.5.

5.5 DDR from heterogeneous discounting

Table 5.6 shows the results from the Gollier heterogeneous discounting. It should be noted that constant PRTP discounting schemes in the table have the same **mean value, not the same initial PRTPs.** This is because the mean value of PRTP distribution is the crucial feature of the heterogeneous discounting scheme, which is somewhat different from previous schemes.

The SCC numbers from Gollier heterogeneous discounting are mostly negative, mainly due to the high PRTPs used. But it is perhaps not the numbers that deserve the most attention, but the percentage increases of SCC. The increases are not sensitive to the mean of PRTPs. This can be seen from the results of a lower mean value of 3%- the magnitude of the increase is roughly the same for all three mean values tested.

Table 5.5 SCC from Gollier heterogeneous discounting

Discounting Scheme	*SCC ($/tC)*
6.4% as mean of PRTPs	-$1.51
Constant 6.4% PRTP	-$3.48
% of SCC increased	57%
3.0% as mean of PRTPs	-$0.65
Constant 3.0% PRTP	-$2.34
% of SCC increased	72%

5.6 Summary of the results

Summing up the results from the above 10 combinations of DDR schemes, the first generalization is that declining discount rates increase the value of SCC, as expected.

Looking at the magnitude of the increases, the range of percentage increases lies between 10% to 4100% without equity weighting, and between 26.9% and 5919% with equity weighting. Among the five schemes, the Weitzman discounting schemes (both the step decline and the gamma discounting) cause the most increases in SCC- the SCC is increased by over 40 times. This implies that FUND 2.8 is most sensitive to the Weitzman discounting, among five schemes.

Looking at the dollar value of the SCC, the upper and lower boundary without equity weighting is: $185/tC and $-2.10/tC (£128/tC and -£1.4/tC). If equity weighting is taken into account, the ranges extends to between \$537/tC and \$-2.25/tC (£370/tC and -£1.6/tC) (equity-weighted results are not listed above). Equity weighting both increases the upper boundary and decreases the lower boundary.

5.7 Discussion

5.7.1 The effects of DDR

In Pearce (2003), it is suggested that time-varying discount rates would raise the SCC boundary by about 80%, much according to Newell and Pizer (2003)'s preferred results. However, the results of the sensitivity studies in this paper suggest that the uncertainty boundary is even larger, especially in the case of the Weitzman discounting, where the DDR raises the estimate by over 40 times. Although this result is based on a different DDR scheme, the study here at least implies that such a large boundary is possible under sensible assumptions (the Weitzman rationale is widely accepted).

Second, the sensitivity studies here show that disaggregating the Ramsey equation can provide useful insights into the effects of DDR. For example, the Gollier growth uncertainty discounting and heterogeneous discounting both deal with a specific element in the Ramsey equation.

Third, although DDR raises the value of SCC, it doesn't push the SCC to a very high level-without equity weighting, only one number out of the ten is larger than \$100/tC. For instance, the UK Green Book discounting scheme raises the value of SCC by 170%, but the SCC number is still much smaller than the £70/tC recommended in the UK DEFRA (2002) paper. The only scheme that gives a higher number than the £70/tC is the Gollier scheme based on uncertainties of growth, combined with a low constant PRTP of 0%. Of course, this result is also subject to the underlying model.

Fourth, the implementation of heterogeneous discounting produces negative SCC numbers because of the survey results on PRTPs in the world. The results reveal that the PRTPs implied by monetary policies in those countries vary to a great extent and many PRTP's are much higher than suggested in the economics literature. This counter-intuitive result is not conclusive. But if international policy-making bodies are to listen to the actual preferences of the country citizens, such a possibility should be taken into account.

5.7.2 The practical implications of DDR

The first practical implication of DDR is whether policies on climate change will pass a cost-benefit analysis. The discussion here focuses on results without equity weighting.

Take the UK's commitment to the Kyoto Protocol for example. According to Pearce (2003), the marginal cost of control under the commitment is £45/tC. If ancillary benefits are not considered, then there are four combinations that would make this commitment pass a CBA, *viz.* the Weitzman scheme and the Gollier scheme based on growth uncertainty, combined with a PRTP of 0%. Surprisingly, although all the ten DDR combinations have significant effects on increasing the numbers of SCC, only a few of them would justify the Kyoto commitment in terms of economic efficiency. Of course, the marginal control cost of £45/tC might also be an overestimate. If the 'flexibility mechanisms' to control emissions (e.g. emissions trading) are adopted, the cost might come down.

If ancillary benefits are included, and are assumed to be £35/tC (Pearce, 2003), then three more combinations would make the avoided damage sufficient to outstrip the control cost.

The second policy implication of the studies here is that policy-making can choose the element of major concern to decide the time profile of discount rates, if DDR is to be applied. For example, if uncertainty about future growth dominates, then Gollier's scheme from growth uncertainty can be adopted. If it's more important to reconcile the uncertainty (or differences) in PRTP, then the heterogeneous discounting can be used. These two uncertainties can even be combined to create new schemes, if both elements are of concern. If underlying elements are not to be concerned, then the Green Book discounting and Weitzman discounting offer a good choice.

A caution to note is the time consistency problem of DDR, which has been discussed in 3.5.5. This is again the problem that policy makers should bear in mind. Although Gollier's heterogeneous discounting seems to provide a way around this problem, it is conditional on the social planner in each generation allocating welfare according to the same efficiency principle and that the allocation in previous generation has an effect on later generation. This is a condition very difficult to guarantee in reality. Therefore, the caveat here is that DDR should not be carried out in a naïve way.

6. Conclusions and areas for further research

6.1 Conclusions

Several conclusions can be gathered from the sensitivity studies in previous chapters.

First of all, as expected, declining discount rates increase the value of SCC. This has been tested in several other Integrated Assessment Models including the RICE model. The extensive sensitivity test here again confirms this in another peer-reviewed model, FUND 2.8.

Second, the increase of SCC by DDR can vary a lot depending on the schemes. The range of increases obtained in this paper is between 10% and 4100% without equity weighting. Some increases in the studies here are substantially higher than those in the literature, such as the Weitzman scheme. This suggests FUND is most sensitive to Weitzman discounting.

The third overall conclusion is that uncertainty in discounting alone does not drive up the SCC to a very high level, e.g. the £70/tC suggested in Clarkson and Deyes (2002). Except for one combination, the highest number of SCC without equity weighting does not exceed £60/tC. This has to do with the discounting schemes and the damage profile in FUND 2.8, which some critiques regard as being optimistic about adaptation.

The novel way of implementing Gollier's heterogeneous discounting results in negative SCC, i.e. benefits, although the increase of SCC is still significant. This implies a probability for low impacts even under DDR schemes.

The sensitivity studies have provided useful information for practical policy making, e.g. CBA studies of climate change policies. It also provides options for policy makers to focus on specific elements within the social discounting functions, such as growth uncertainties or heterogeneity of time preferences. One caveat for policy making is the potential time inconsistency of DDR, which implies that policies that use DDR should not be applied naively.

6.2 Areas for further research

One of the highlights of this paper is the implementation of Gollier's heterogeneous discounting scheme. However, as mentioned in 4.3.3, the method to derive a mean PRTP for the world is second-best in terms of data source. Therefore, further research into how this can be improved is needed, as well as taking an even sophisticated approach towards to distribution of PRTPs in the world.

Second, if DDR is to have even more pronounced effects, it should be implemented to models with even longer time horizons than the one in FUND 2.8. With the progress of climate science and IAM, such an implementation will become more realistic.

Appendix 1: Hyperbolic Discounting

The discount factor based on Lowenstein and Prelec (1992) is:

$$d_t = \frac{1}{(1+kt)^{h/k}} \qquad \text{(A1.1)}$$

An empirically estimation of the parameters in the above equation is: k = 4, and h = 1. When t =1, d_t = 0.669 and the discount rate necessary to get the same discount factor can be calculated by solving the following equation:

$$d_t = \frac{1}{(1+s)^t} = \frac{1}{1+s} = 0.669 \qquad \text{(A1.2)}$$

And the result is s = 49.5%, much higher than the ones used in the literature, although it falls rapidly as time goes by.

Appendix 2: Gollier's model

Gollier's model contains two periods: period one for the near future and period two for the distant future. The consumption growth in each period is denoted as $\tilde{g}_t$, which is uncertain and follows a random walk pattern. The key point is that in an uncertain world, the decision-maker must construct expectations about the future in order to determine the optimal discount rate. The decision-maker's objective is as follows

$$\max U(z_t)\sum_{t=0}^{\infty}\beta^t E\left[u_t(\tilde{z}_t)\right] \quad \text{(A2.1)}$$

where *E[.]* represents the expectation operator, z_t represents the uncertain consumption level in period *t*. Then the short-term gross interest rate that will prevail in period *t =1* as a function of consumption *c* is as follows:

$$\rho(c) = \frac{u'(c_0\tilde{g}_1)}{\beta E[u'(c_0\tilde{g}_1\tilde{g}_2)]} \quad \text{(A2.2)}$$

where $\tilde{g}_t$ is the uncertain growth rate of consumption in period *t*.

To see whether *ρ(c)* is declining over time, the first derivative of (A2.2) with respect to consumption is used, which is stated as:

$$z\rho'(c) = \rho(c)[R(c\tilde{g}_2) - R(c)] \quad \text{(A2.3)}$$

where $R(c)$ represents the index of relative risk aversion of individuals (agents) in period one, and $R(c\tilde{g}_2)$ the equivalent term in period two.

It's clear that when $R(c\tilde{g}_2) < R(c)$, (A2.3) is negative, i.e. *ρ(c)* is declining over time. If we assume there is no risk of recession in the economy, then $c\tilde{g}_2 > c$. If R(c) decreases with wealth, then $R(c\tilde{g}_2) < R(c)$ will be true. In other words, when there is no risk of recession both in the near and distant future, and if risk aversion decreases with wealth, the discount rate is declining over time.

When there is a risk of recession, the concept of relative prudence is introduced as:

$$P(c) = -c\frac{u'''(c)}{u''(c)} \tag{A2.4}$$

It is proved that when there is risk of recession only in the long run (i.e. in period two), the necessary conditions for the discount rate to decline over time are: *R(c)* decreases with wealth and *P(c)* increases with wealth. When recession is possible to happen in both periods, the first and second derivative of P(c) are used to place more restrictions on the conditions to generate DDR, which corresponds to the 4th and 5th derivative of the utility function with respect to consumption.

Appendix 3: Chichilnisky's approach

Chichilnisky's criterion can be represented in the following objective function (Heal, 1998):

$$\max\left[\alpha\int_0^t u\left(c_t,q_t\right)e^{-\rho t}dt+(1-\alpha)\lim_{t\to\infty}u\left(c_t,q_t\right)\right] \qquad \text{(A3.1)}$$

where the utility function *u(.)* has two arguments: consumption *(c_t)* and resource stock *(q_t)* at each time period *t*. $e^{-\rho t}$ is the conventional exponential discount factor. The first term in (A3.1) means that up to time period *t* in the near future, utilities will be discounted in a conventional way. And the second term means that the utilities in the rest periods after *t* will not be discounted. $\alpha\in(0,1)$ can be interpreted as the weights that the decision-maker places on the near and the distant future, i.e. the two components in Chichilnisky's axiom. The technical difficulty of applying this formula, as Dasgupta (2001) points out, is choosing the 'switching date'- *t*. Dasgupta (2001) shows that it is always possible to improve aggregate wellbeing by postponing the switching date. Also, this formula gives no indication of choosing a value for α- the weight on the 'near' future.

Appendix 4: The Li and Löfgren approach

The objective utility function in Li and Löfgren (2000) is:

$$U = \alpha U_1 + (1-\alpha)U_2 = \int_0^\infty u(c_t, s_t)\, p(t)\, dt \tag{A4.1}$$

where

$$U_1 = \int_0^\infty u(c_t, s_t) \exp(-\rho t)\, dt \tag{A4.2}$$

is the objective utility function of the utilitarian and

$$U_2 = \lim \int_0^\infty u(c_t, s_t) \exp(-\delta t)\, dt \tag{A4.3}$$

is the objective utility function of the conservationist. And *p(t)* is the time-varying discount factor.

The implied utility discount rate from (A4.1) is given by:

$$a(t) = \frac{-\ln\{(1-\alpha)\exp(-\delta t) + \alpha \exp(-\rho t)\}}{t} \tag{A4.4}$$

The key technical problem here then, is to choose a value for $\alpha \in (0,1)$, the weight placed on the utilitarian and the conservationist.

Appendix 5: Details on the result of the PRTP survey

Region/Country	*Number of Observations*	*Mean*
ANZ(2)		
Australia	26	8
New Zealand	42	7
CAM(2)		
Costa Rica	42	9
Belize	26	15
CAN (1)		
Canada	36	5
CHI(1)		
China	13	-2
EEU(7)		
Albania	11	3
Croatia	9	0
Hungary	18	13
Poland	12	14
Czech Republic	10	6
Slovenia	9	6
FSU(2)		
Latvia	10	2
Russia	8	-2
JPK(2)		
Japan	42	0
Korea	42	4
LAM(9)		
Bolivia	40	12
Chile	10	4
Colombia	42	17
Dominica	9	7
Ecuador	33	14
Guyana	37	12
Paraguay	15	17
Peru	42	7
Venezuela	42	13
MAF(5)		
Algeria	29	5
Egypt	42	4

Region/Country	Number of Observations	Mean
Libya	27	1
Morocco	42	2
Tunisia	35	4
MDE(5)		
Isreal	21	-7
Jordan	27	4
Kuwait	28	8
Lebanon	14	15
Syrian	42	2
Yemen	8	13
SAS(5)		
Bangladesh	32	7
India	40	5
Nepal	27	8
Pakistan	42	6
Sri Lanka	40	7
SEA(6)		
Indonesia	13	10
Lao	11	16
Malaysia	36	0
Myanmar	11	3
Philippines	42	6
Thailand	27	4
Vietnam	7	3
SIS(15)		
Antigua and Barbuda	9	6
Bahamas	31	7
Barbados	26	9
Cyprus	27	0
Fiji	29	5
Grenada	9	5
Malta	34	0
Netherlands Antilles	5	10
Papua New Guinea	20	9
St. Kitts and Nevis	9	5
St. Lucia	9	9
St. Vincent & Grens.	9	7
Trinidad and Tobago	37	5
Vanuatu	5	9
USA(1)		
USA	42	3
WEU(16)		
Austria	38	2

Region/Country	Number of Observations	Mean
Belgium	38	4
Denmark	42	4
Finland	38	4
France	28	4
Germany	27	2
Greece	40	8
Iceland	42	7
Italy	38	5
Netherlands	33	3
Norway	42	3
Portugal	38	5
Spain	38	4
Sweden	38	4
Switzerland	42	2
UK	20	6
SSA(34)		
Benin	42	6
Botswana	27	4
Burkina Faso	42	5
Burundi	29	8
Cameroon	35	6
Central Africa	35	8
Chad	35	7
Congo, Republic of	35	6
Cote d'Ivoire	42	5
Equatorial Guinea	17	-5
Gabon	35	5
Gambia, The	29	10
Ghana	42	12
Guinea	16	14
Guinea-Bissau	13	9
Kenya	33	11
Lesotho	23	10
Malawi	39	10
Mali	35	7
Mauritius	18	6
Mozambique	9	8
Namibia	12	13
Niger	42	8
Nigeria	42	7
Rwanda	39	7
Senegal	42	7
Seychelles	22	1

Region/Country	Number of Observations	Mean
South Africa	42	8
Swaziland	27	9
Tanzania	14	16
Togo	42	5
Uganda	20	6
Zambia	38	6
Zimbabwe	39	9

Bibliography

International Monetary Fund (2003). International Financial Statistics, ESDS International, University of Manchester.

World Bank (2003). World Development Indicators, ESDS International, University of Manchester.

The Russia Journal (2004). Top Scientists tell Putin to kill Kyoto. http://www.russiajournal.com/news/cnews-article.shtml?nd=43814 (2004-08-20)

Anthoff, D. (2004). Social cost of carbon:equity weighting. MSc thesis, Environmental Change Institute, University of Oxford. Oxford,

Broome, J. (1992). Counting the cost of global warming: A report to the Economic and Social Research Council. Cambridge, White Horse Press.

Chichilnisky, G. (1997). "What is sustainable development." Land Economics **73**: 467-491.

Clarkson, R. and K. Deyes (2002). Estimating the social cost of carbon emissions. Government Economic Service Working Paper. London, HM Treasury.

Cline, W. R. (1992). The Economics of Climate Change. Washington DC, Institute for International Economics.

Cropper, M. Ayded, S. and P. Portney (1992). "Rates of time preference for saving lives" American Economic Review **82**: 469-72.

Cowell, F. A. and K. Gardiner (1999). Welfare Weights, STICERD, London School of Economics.

Dasgupta, P. (2001). Human Wellbeing and the Natural Environment. Oxford, Oxford University Press.

DEFRA (Department of Environment, Food and Rural Affairs) (2003). The Social Cost of Carbon Review Background Paper. London.

DeGroot, M. H. and M. J. Schervish (2002). Probability and Statistics (3rd ed.). Boston, Addison Wesley.

Eyre, N. et al. (1999). 'Global Warming Damages', Final Report of the ExternE Global Warming Sub-Task(Sept.98),DGXII. Brussels, European Commission.

Fankhauser, S. (1995). Valuing climate change: The economics of the greenhouse. London, Earthscan Publications Limited.

Federick, S., G. Loewenstein, et al. (2002). "Time Discounting and Time Preference: A Critical Review." Journal of Economic Literature **XL**: 351-401.

Gollier, C. (2002a). "Time horizon and the discount rate." Journal of Economic Theory **107**: 463-473.

Gollier, C. (2002b). "Discounting an Uncertain Future." Journal of Public Economics **85**(2): 149-166.

Gollier, C. and R. Zeckhauser (2003). Collective investment decision making with heterogeneous time preferences. NBER Working Paper 9629. Cambridge, Massachusetts, National Bureau of Economic Research (NBER).

Heal, G. (1998). Valuing the Future: Economic Theory and Sustainability. New York, Columbia University Press.

Hepburn, C. (2003). Hyperbolic discounting and resource collapse, Oxford University Department of Economics Working Paper.

IPCC (1996). Economic and Social Dimensions of Climate Change: Summary for Policy Makers. Climate Change: 1995. New York, Cambridge Unviersity Press.

IPCC (2001). Climate Change 2001: The scientific basis. New York, Cambridge University Press.

Judge, G. G., R. C. Hill, et al. (1982). Introduction to the Theory and Practice of Econometrics. New York, John Wiley.

Kimball, M. S. (1990). "Precautionary Saving in the Small and in the Large." Econometrica **58**: 53-73.

Koopmans, T. (1960). "Stationary Ordinal Utility and Impatience." Econometrica **28**: 287-309.

Li, C. Z. and K. G. Lofgren (2000). "Renewable Resources and Economic Sustainability: A Dynamic Analysis with Heterogeneous Time Preferences." Journal of Environmental Economics and Management **40**: 236-250.

Lind, R. C. (1982). Discounting for Time and Risk in Energy Policy. Baltimore, Johns Hopkins University Press.

Loewenstein, G. and D. Prelec (1992). "Anomalies in Intertemporal Choice: Evidence and an Interpretation." Quarterly Journal of Economics **107**: 573-597.

Newell, R. G. and W. A. Pizer (2003). "Discounting the distant future: How much do uncertain rates increase valuations?" Journal of Environmental Economics and Management **46**: 52-71.

Nordhaus, W. D. (1990). "To Slow or Not to Slow: The Economics of the Greenhouse Effect." The Economic Journal **101**(407): 920-937.

Nordhaus, W. D. (1994). Managing the Global Commons: the Economics of Climate Change. Cambridge, MA, MIT Press.

Nordhaus, W. D. and J. Boyer (2000). Warming the world: Economic models of global warming. Cambridge, MIT Press.

Ogaki, M. and Q. Zhang (2000). Risk Sharing in Village India: the Rule of Decreasing Relative Risk Aversion, Ohio State University working paper.

Olson, M. and M. Bailey (1981). "Positive Time Preference." Journal of Political Economy **89**(1): 1-25.

OXERA Consulting Ltd. (2002). A social time preference rate for use in long-term discounting. Report to Office of the Deputy Prime Minister, Department for Transport, and DEFRA.

Pearce, D. W., W. R. Cline, et al. (1996). The social costs of climate change: Greenhouse damage and the benefits of control. Intergovernmental Panel on Climate Change, Climate Change 1995: Economic and Social Dimensions of Climate Change. Cambridge, Cambridge University Press**:** 183–224.

Pearce, D. and D. Ulph (1999). A Social Discount Rate for the United Kingdom. Economics and the Environment: Essays in Ecological Economics and Sustainable Development. D. Pearce. Cheltenham, Edward Elgar**:** 268-285.

Pearce, D. (2003). "Social Cost of Carbon and its policy implications." Oxford Review of Economic Policy **19**(3): 362-384.

Ramsey, F. P. (1928). "A Mathematical Theory of Saving." Economic Journal **38**: 543-559.

Rawls, J. (1972). A Theory of Justice. Oxford, Oxford University Press.
Samuelson, P. (1937). "A Note on Measurement of Utility." Review of Economic Studies **4**: 155-161.

Strotz, R. (1956). "Myopia and inconsistency in dynamic utility maximisation." Review of Economic Studies **23**: 163-180.

Thomas, H. A. (1963). "Animal Farm: A Mathematical Model for the Discussion of Social Standards for Control of Environment." Quarterly Journal of Economics **7**(1): 143-148.

Tol, R. (2003a). "Emission abatement versus development as strategies to reduce vulnerability to climate change: an application of FUND (working paper)."

Tol, R. (2003b). "The marginal costs of carbon dioxide emissions (working paper)."

Tol, R. and T. Heinzow (2003). Estimates of the external and sustainability costs of climate change.Centre for Marine and Climate Research working paper FNU-32, Hamburg, Germany, Hamburg University.

Tol, R. (2004). "The Marginal Damage Cost of Carbon Dioxide Emissions: An Assessment of the Uncertainties (working paper)." Centre for Marine and Climate Research working paper, Hamburg, Germany, Hamburg University.

H.M.Treasury (2003). The Greenbook Appraisal and Evaluation in Central Government. London, TSO.

Warner, J. T. and S. Pleeter (2001). "The Personal Discount Rate: Evidence from Military Downsizing Programs." American Economic Review **91**(1): 33-53.

Weitzman, M. (1998). "Why the far distant future should be discounted at its lowest possible rate." Journal of Environmental Economics and Management **36**: 201-208.

Weitzman, M. (2001). "Gamma Discounting." American Economic Review **91**(1): 261-271.

Wilson, R. (1968). "The theory of syndicates." Econometrica **36**: 113-132.